

Intracultural diversity in a model of social dynamics

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Abstract

We study the consequences of introducing individual nonconformity in social interactions, based on Axelrod's model for the dissemination of culture. A constraint on the number of situations in which interaction may take place is introduced in order to lift the unavoidable homogeneity present in the final configurations arising in Axelrod's related models. The inclusion of this constraint leads to the occurrence of complex patterns of intracultural diversity whose statistical properties and spatial distribution are characterized by means of the concepts of cultural affinity and cultural cline. It is found that the relevant quantity that determines the properties of intracultural diversity is given by the fraction of cultural features that characterizes the cultural nonconformity of individuals.

Key words: Social dynamics; Cultural diversity

1 Introduction.

Recently, many dynamical models, inspired by analogies with physical systems, have been proposed to describe a variety of phenomena occurring in social dynamics [1,2,3,4]. Examples include the emergence of cooperation and self-organization, propagation of information and epidemics, opinion formation, economic exchanges and evolution of social structures. In this context, Axelrod's model [5] for the dissemination of culture among interacting agents in a social system has attracted much attention among physicists.

The concept of culture introduced by Axelrod refers to a set of individual features or attributes that are subject to social influence. Agents can interact with their neighbors in the system according to the cultural similarities that they share. From the point of view of statistical physics, this model is appealing because it exhibits a nonequilibrium transition between an ordered final frozen

state (a global homogeneous culture) and a disordered (culturally fragmented) one [6,7,8,9]. Several extensions of this model have recently been investigated. For example, cultural drift has been modeled as noise acting on the frozen disordered configurations [10]. The effects of mass media has been considered as external [11] or autonomous [12,13] influences acting on the system. The role of the topology of the social network of interactions have also been addressed [8,14,15]. Other extensions include the consideration of quantitative instead of qualitative values for the cultural traits [16]. These studies have revealed that Axelrod's model is robust in the sense that its main properties persists in all those cases. In particular, the final frozen states invariably consist of one or more homogeneous cultural groups.

In this paper, we introduce a constraint on the number of situations in which interaction may take place, in order to lift the unavoidable homogeneity in the final states of the above models. Our model is motivated by the idea that generally individuals tend to maintain a minimum degree of identity by keeping some cultural features different from those of their neighbors. This restriction naturally leads to the persistence of complex patterns of diversity in the cultural groups present in the final state of the system.

The model is explained in Section 2. In Section 3, the results of numerical simulations are presented, showing the patterns of diversity in the final frozen states. The statistical properties that characterize intracultural diversity are calculated in Section 4. Conclusions are presented in Section 5.

2 Axelrod's model with intracultural diversity.

Axelrod's model [5] considers a square lattice network of $N = L^2$ elements with open boundaries and nearest neighbor interactions. The state of element i is given by a cultural vector of F components (cultural features) $(s_{i1}, s_{i2}, \dots, s_{iF})$. Each component s_{if} can adopt any of q integer values (cultural traits) in the set $\{1, \dots, q\}$. Starting from a random initial state the network evolves at each time step following these simple rules: i) An element i and one of its four neighbors j is selected at random. ii) If the overlap, defined as $\omega(i, j) = \sum_{f=1}^F \delta_{s_{if}, s_{jf}}$, (number of shared features) is in the range $0 < \omega(i, j) < F$, the pair (i, j) is said to be active with a probability of interaction equal to $\omega(i, j)/F$. iii) In case of interaction, one of the unshared features k is selected at random and element i adopts the trait s_{jk} , thus decreasing in one unit the overlap of the pair (i, j) .

In any finite network the dynamics settles into a frozen state, characterized by either $\omega(i, j) = 0$ or $\omega(i, j) = F$, $\forall i, j$. Homogeneous or monocultural states correspond to $\omega(i, j) = F$, $\forall i, j$, and obviously there are q^F possible configura-

rations of this state. Inhomogeneous or multicultural states consist of two or more homogeneous domains interconnected by elements with zero overlap. A domain, or a cultural region, is a set of contiguous sites with identical cultural traits. Castellano et al. [6] demonstrated that the final states of the system experience a transition from ordered states (homogeneous culture) for $q < q_c$ to disordered states (cultural fragmented) for $q > q_c$, where q_c is a critical value that depends on F .

In order to allow for diversity we introduce a parameter F_d such that a pair (i, j) is considered active if the overlap is in the range $0 < \omega(i, j) < F - F_d$, with $0 < F_d < F$. There is no restriction on which of the F_d features cannot be exchanged by an active pair. The case $F_d = 0$ recovers the original Axelrod's model, whereas $F_d = F$ results in frozen configurations for all possible initial configurations. The number of possible frozen states is the number of configurations in which neighbors cannot longer interact; thus increasing F_d results in an increase of this number. The parameter F_d reduces the number of situations in which interactions may take place. In the context of social dynamics, the ratio F_d/F can be seen as a measure of individual nonconformity.

A cultural region is a set of contiguous sites that possess the same cultural vector, whereas a cultural zone is defined as a set of contiguous sites that share one or more cultural traits; elements in a cultural zone are said to have a "compatible" culture [5]. Cultural zones appear as transient states in the original Axelrod's model, but in the final state only cultural regions are present. When $F_d > 0$, cultural zones will usually be present in the final state because then contiguous sites have an overlap $\omega(i, j) \geq F - F_d$. The model can be modified by fixing in advance a subset of features that the elements of a cultural zone must share in the final state.

3 Numerical Results

As an example of the effects resulting from the inclusion of the parameter F_d in Axelrod's model, we shall consider a system of size $N = 20 \times 20$ with $F = 11$, and $q = 10$, starting from random initial conditions. For $F_d = 0$, the system converges to a homogeneous state, i.e., a single cultural region, since $q \ll q_c \sim 60$. For $F_d = 1$ the final state consists of a single cultural zone possessing few surviving traits. We denote by $K_f(s)$ the number of times that the trait value s appears in the f th feature of the cultural vectors in the system. That is, $K_f(s) = \sum_{i=1}^N \delta_{s_{if}, s}$. For a particular realization in a system of size $N = 20 \times 20$, Table 1 shows $K_f(s)$ for the final state. The first row in Table 1 shows that $K_1(1) = N$ and for the remaining traits $K_{f=1}(s \neq 1) = 0$; that is, all the 400 cultural vectors have reached the value 1 in their first feature. Note that all 400 elements also share the traits associated to features

$f = 6, 9$, and 10 . In features $f = 3, 4, 7, 8$, and 11 only 2 different values of traits survive; while in features $f = 2$ and 5 there appear 3 different values of traits. The fraction of trait values that disappear during the evolution towards the final state of the system is $\sum_{f,s} \delta_{K_f(s),0} / (f \times (q + 1))$. Therefore, for the realization in Table 1, about 82% of all the trait values existing in the cultural vectors at the beginning have disappeared in the final state of the system.

In the final state there are 47 different cultural vectors in the system. Unexpectedly, the abundance of these vectors follows a nonuniform distribution. Table 2 shows the distribution of the 15 most abundant cultural vectors in the final state. These vectors are ranked according to their abundance. The abundance of the vector of rank R is denoted by N_R and it is also indicated in Table 2. We define the fraction of elements having the cultural vector with rank R as $C_R = N_R/N$. Note that about half of the elements in the final state share its cultural vector among the seven most common vectors.

Figure 1 shows the pattern of the final state of the system. The labels indicate the rank of the cultural vector corresponding to each site. Contiguous sites having identical cultural vectors are joined by lines. Elements with identical cultural vectors tend to form domains, in spite that neighbors with overlap $F - 1$ do not interact.

The number of surviving traits monotonically decreases during the evolution of the system toward its final state. When $F_d = 0$ the number of surviving traits in the final state is F if $q < q_c$ (all elements are identical and therefore there is one trait per feature). As shown above, for $F_d > 0$ the number of surviving traits in the final state is greater than F but much smaller than its maximum possible value of $F \times q$. On the other hand, the size of the largest domain is equal to N for $F_d = 0$ and $q < q_c$, while for $F_d > 0$ the largest group involves only a fraction of the elements in the system. Figure 2 shows the evolution of both, the number of surviving traits and the fraction of elements having the most abundant cultural vector, $C_1 = N_1/N$, in a system of size $N = 120 \times 120$, for $F_d = 0$ and $F_d = 1$.

4 Statistical properties

Figure 3 shows the average fraction of elements with the most abundant vector $\langle C_1 \rangle$ as a function of q , for several values of the parameter F_d in a system of size $N = 40 \times 40$ and $F = 11$. For $F_d = 0$, there exist a critical value q_c at which the order parameter $\langle C_1 \rangle$ exhibits an abrupt transition from a homogeneous, monocultural state, characterized by $\langle C_1 \rangle = 1$, to a disordered, multicultural state, where $\langle C_1 \rangle \ll 1$ [6]. The value q_c is not very sensitive to the variation of the parameter F_d . For $F_d > 0$ and $q < q_c$, the value of $\langle C_1 \rangle$ is less than

1, indicating the presence of intracultural diversity in the single surviving cultural zone (there are no neighbors with zero overlap).

Figure 4 shows the average fraction of cultural vectors $\langle C_R \rangle$ by rank R , for different system sizes. Each curve is the average over 50 realizations of initial conditions. For the parameter values used in Fig. 4 ($q \ll q_c$) there is a single cultural zone in the final state but several cultural vectors survive having a distribution that depends on the system size. The dispersion on each curve is larger for low and for high values of R than for intermediate values of the rank. The dispersion for low values of R reflects the competition between the more abundant vectors during the evolution towards the frozen state. On the other hand, the dispersion for large values of R is mostly due to the fluctuations on the total number of different vectors present in the final state.

The frozen patterns in a cultural zone are complex, as shown in Figure 1. The cultural diversity can be characterized in terms of the distribution of the cultural affinity between any two elements i and j in the system, defined as $A(i, j) = \omega(i, j)/F$. Figure 5 shows the average distribution of the cultural affinities $A(i, j)$ of all pairs of elements in the system, averaged over 10 realizations. The three curves on each panel in Fig. 5 correspond to three different values of q . In the case that $F_d = 0$, the distribution for $q < q_c$ consists of a single peak at the value $A(i, j) = 1$, corresponding a homogeneous state, while for $q > q_c$ a second peak appears at $A(i, j) = 0$, reflecting the presence of multiple domains. As shown in Fig. 5, for $F_d > 0$ intracultural diversity is manifested as a wide spectrum of cultural affinities present in the system. As F_d increases, the distribution of cultural affinities shifts towards $A(i, j) = 0$, reflecting the increase of intracultural diversity within the cultural zones.

The spatial distribution of cultural diversity can be characterized by the average shell affinity $S(r)$ defined as

$$S(r) = \frac{1}{N \times n(r)} \sum_{i=1}^N \sum_{j \in \rho(i, r)} A(i, j), \quad (1)$$

where $\rho(i, r)$ is the set of elements in a square shell of radius r centered in element i (the unit of distance is one site), and $n(r)$ is the number of elements on this shell.

Figure 6 shows $S(r)$ for several values of F_d . The average shell affinity $S(r)$ is well fitted by the relation

$$S(r) \simeq S(1) - \alpha \log r. \quad (2)$$

We find the scaling $S(1) \simeq 1 - F_d/F$ and $\alpha \simeq 1.43F_d/F$ for a wide range

of values of F , F_d and $q < q_c$. The slope $\alpha = -\frac{dS(r)}{d\log(r)}$ characterizes the gradient of intracultural diversity or *cultural cline*. Extrapolation of equation (2) to $S(r_0) = 0$ allows for a definition of a characteristic distance $r_0 = \exp\left[0.7\left(\frac{F}{F_d} - 1\right)\right]$ between two elements that have zero overlap; a concept that can be applied to the study of the spatial distribution of related cultures. These results suggest that the relevant quantity for the description of intracultural diversity is the ratio F_d/F .

5 Conclusions

In order to allow for individual nonconformity in the Axelrod's model for cultural dissemination, we have introduced a parameter F_d that reduces the maximum number of shared features for interaction. The inclusion of this parameter maintains the main features of the Axelrod's model, corresponding to $F_d = 0$. In particular, there is a nonequilibrium transition from a single cultural zone to a multicultural state at about the same critical value q_c at which the Axelrod's model exhibits a transition from a homogeneous, monocultural region, to a multicultural state. However, the addition of parameter F_d set the stage for the occurrence of complex patterns of intracultural diversity in cultural zones. The intracultural diversity associated to the constraint $F_d > 0$ is manifested in the appearance of a distribution of the abundance of cultural vectors by rank inside cultural zones. We found that this distribution is sensitive to the size of the zone, as shown in Figure 4.

We have introduced the concept of cultural affinity between any two elements in order to characterize intracultural diversity in the system. The cultural affinity among all the elements in the system for $F_d > 0$ shows a wide distribution in contrast to the case $F_d = 0$ where the cultural affinity can only take the values 1 when $q < q_c$, or 1 and 0 when $q > q_c$.

The cultural affinity between elements separated by a given distance $S(r)$ has led us to the concept of cultural cline defined as $\alpha = -\frac{dS(r)}{d\log(r)}$. We found that $S(r)$ depends only on the ratio F_d/F since it is well fitted by the relation $S(r) \simeq 1 - F_d/F - 1.43F_d/F \log r$ for a wide range of values of F , F_d and $q < q_c$.

The model presented here may be useful to describe the emergence of cultural gradients such as dialects, gastronomic customs, etc, in geographical areas. In the biological context, this model can also be adapted to the study of phenotype clines. For instance, it have been proposed that the evolution of female mating preferences can greatly amplify large-scale geographic variation in male secondary sexual characters and produce widespread reproductive isolation with no geographic discontinuity [17].

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TABLE 1

 $K_f(s)$: Traits present in the final state

| Feature (f) | Trait (s) | | | | | | | | | |
|-----------------|---------------|-----|-----|-----|----|-----|---|-----|-----|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 0 | 400 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 280 | 63 | 57 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 361 | 0 | 39 | 0 | 0 |
| 4 | 0 | 0 | 225 | 0 | 0 | 175 | 0 | 0 | 0 | 0 |
| 5 | 49 | 0 | 0 | 0 | 36 | 0 | 0 | 0 | 315 | 0 |
| 6 | 0 | 0 | 0 | 400 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 292 | 108 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 132 | 0 | 0 | 0 | 268 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 400 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 400 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 27 | 0 | 0 | 373 | 0 | 0 | 0 | 0 |

TABLE 2

More common vectors in the final state

| rank | occurrence | accumulated fraction | VECTOR |
|------|------------|-------------------------|-----------------------------------|
| 1 | 58 | 0.145 | (1, 1, 5, 2, 8, 3, 0, 5, 7, 5, 5) |
| 2 | 52 | 0.275 | (1, 1, 5, 2, 8, 3, 0, 1, 7, 5, 5) |
| 3 | 28 | 0.345 | (1, 1, 5, 5, 8, 3, 0, 5, 7, 5, 5) |
| 4 | 23 | 0.403 | (1, 1, 5, 5, 8, 3, 1, 5, 7, 5, 5) |
| 5 | 16 | 0.443 | (1, 2, 5, 5, 8, 3, 0, 5, 7, 5, 5) |
| 6 | 14 | 0.478 | (1, 1, 5, 5, 0, 3, 1, 5, 7, 5, 5) |
| 7 | 13 | 0.510 | (1, 1, 5, 2, 8, 3, 1, 5, 7, 5, 5) |
| 8 | 13 | 0.542 | (1, 1, 5, 2, 8, 3, 0, 1, 7, 5, 2) |
| 9 | 12 | 0.573 | (1, 1, 5, 2, 4, 3, 0, 1, 7, 5, 5) |
| 10 | 11 | 0.600 | (1, 2, 5, 5, 8, 3, 1, 5, 7, 5, 5) |
| 11 | 11 | 0.627 | (1, 3, 5, 2, 8, 3, 0, 1, 7, 5, 5) |
| 12 | 10 | 0.652 | (1, 1, 7, 2, 8, 3, 0, 5, 7, 5, 5) |
| 13 | 9 | 0.675 | (1, 3, 5, 2, 4, 3, 0, 1, 7, 5, 5) |
| 14 | 8 | 0.695 | (1, 2, 7, 5, 8, 3, 1, 5, 7, 5, 5) |
| 15 | 8 | 0.715 | (1, 2, 7, 5, 8, 3, 0, 5, 7, 5, 5) |

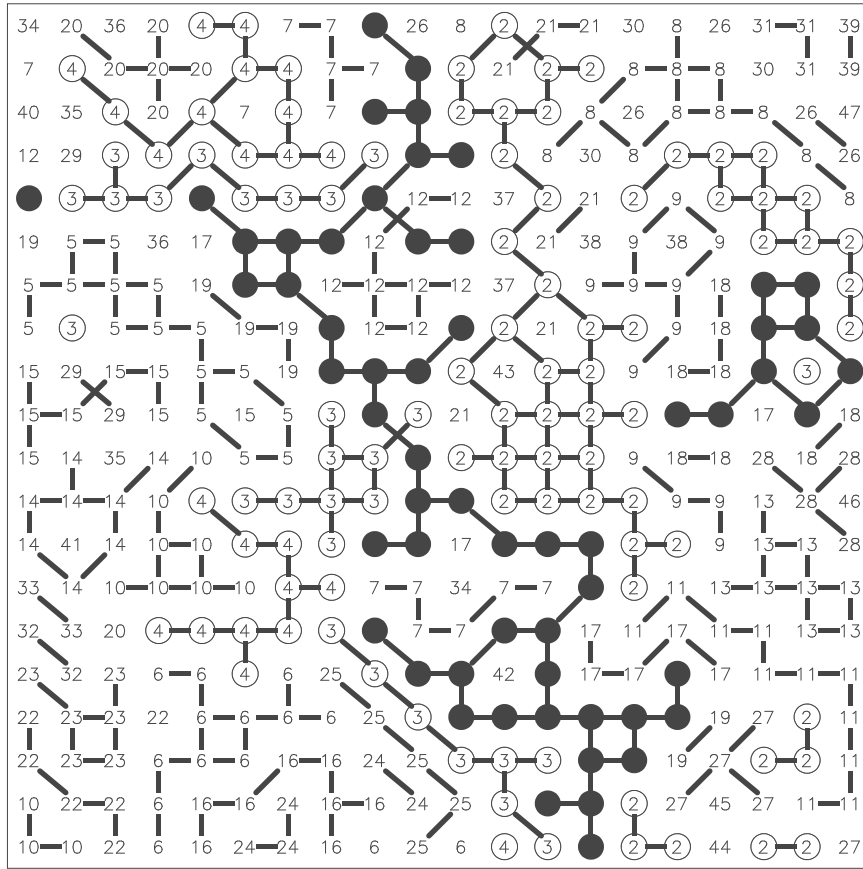


Fig. 1. Spatial pattern corresponding to a final state for a system of size $N = 20 \times 20$ with parameters $F = 11$, $F_d = 1$, and $q = 10$. The numbers on each site indicate the rank of the cultural vector of that site. The most common vectors (rank $R = 1$) are plotted as solid circles. Sites whose vectors have ranks $R = 2, 3, 4$ are plotted as open circles. In order to facilitate the recognition of patterns a line between contiguous sites having identical states are connected by lines.

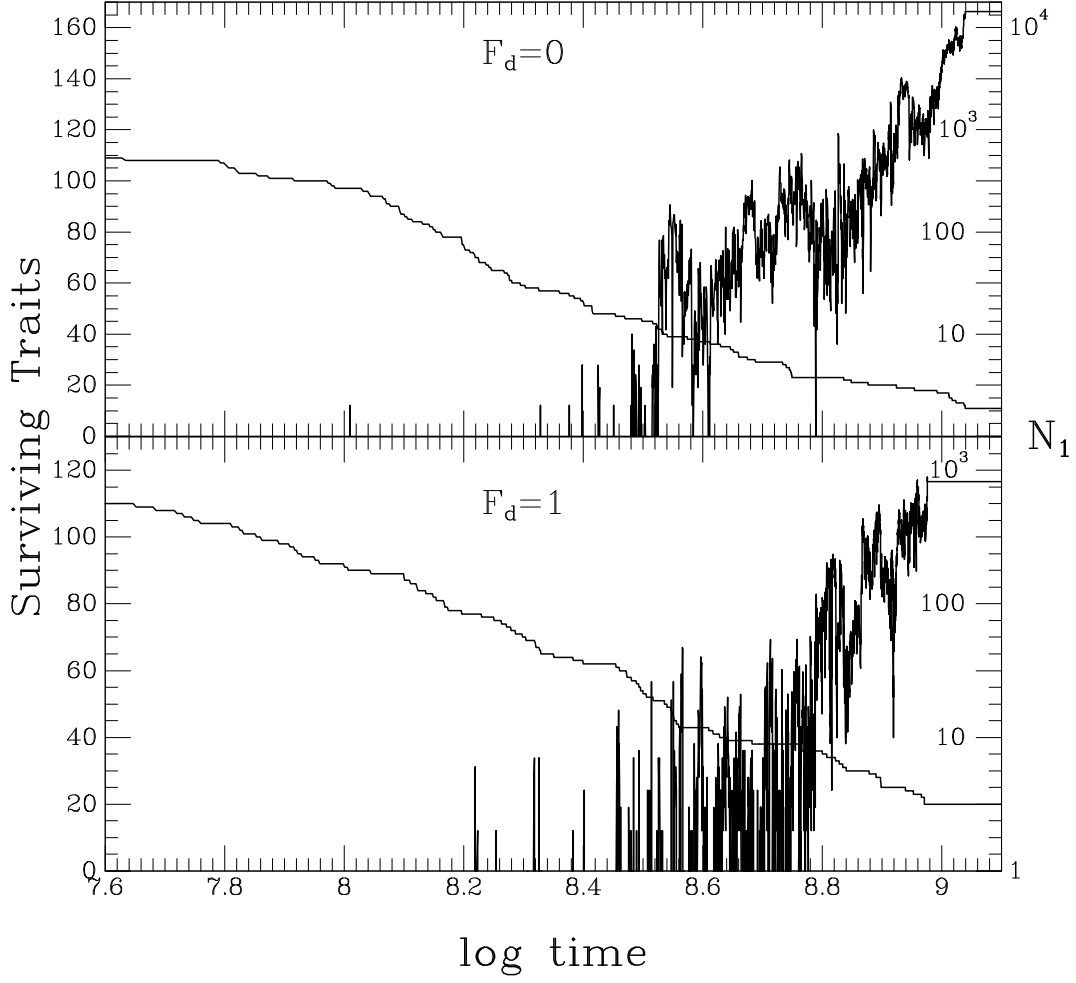


Fig. 2. Evolution for a system of size $N = 120 \times 120$ with parameters $F = 11$ and $q = 10$, for $F_d = 0$ (upper panel) and $F_d = 1$ (lower panel). The monotonically decreasing curve on each panel corresponds to the number of surviving traits (left vertical axis) as a function of time. The fluctuating curve on each panel shows the size of the largest cultural group N_1 (right vertical axis) as a function of time. Time is measured as the number of iterations that result in a trait exchange.

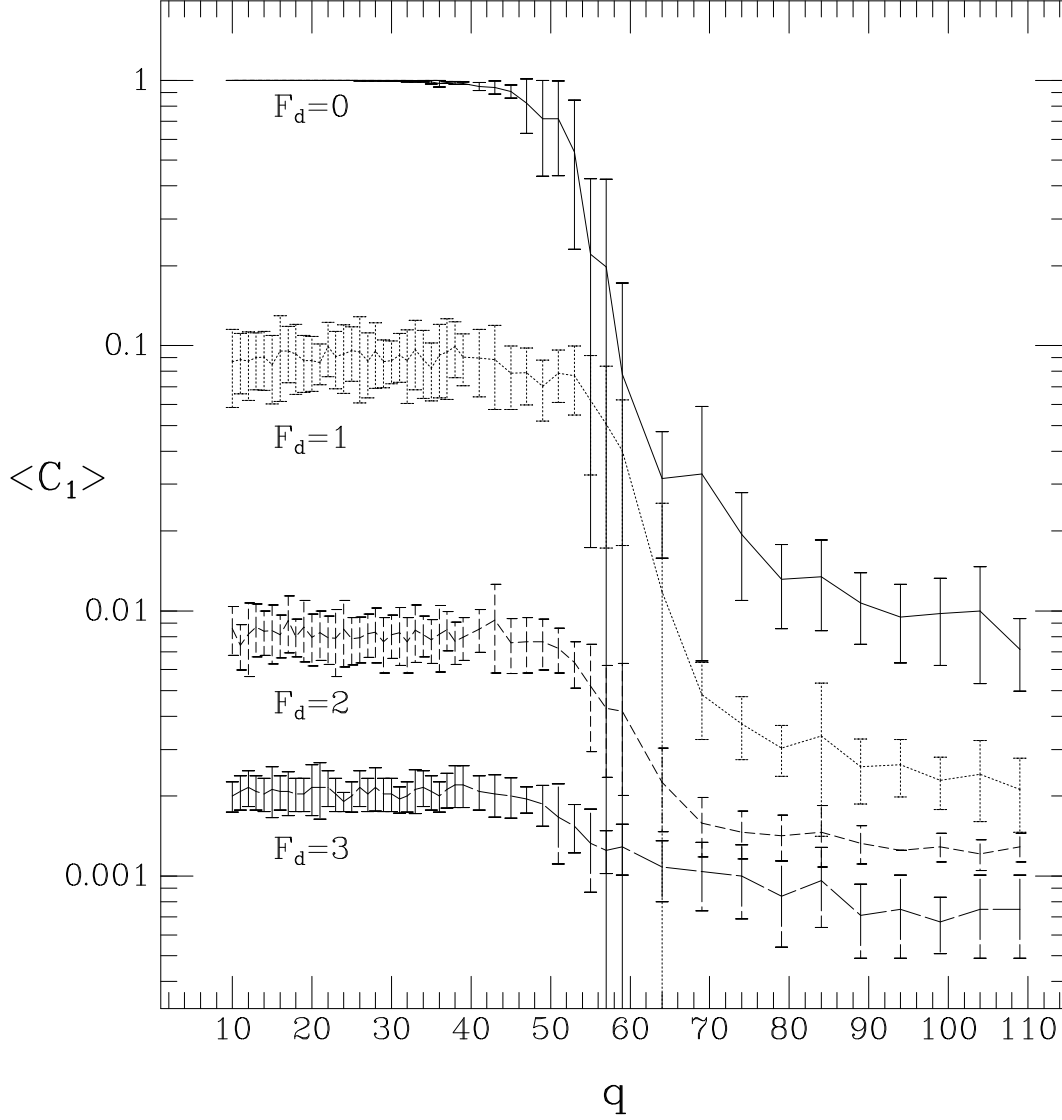


Fig. 3. Average fraction of elements having the most abundant cultural vector, $\langle C_1 \rangle$, as a function of q , for several values of F_d in a system of size $N = 40 \times 40$ and $F = 11$. The label on each curve indicates the value of F_d . Error bars corresponding to one standard deviation are shown on each curve.

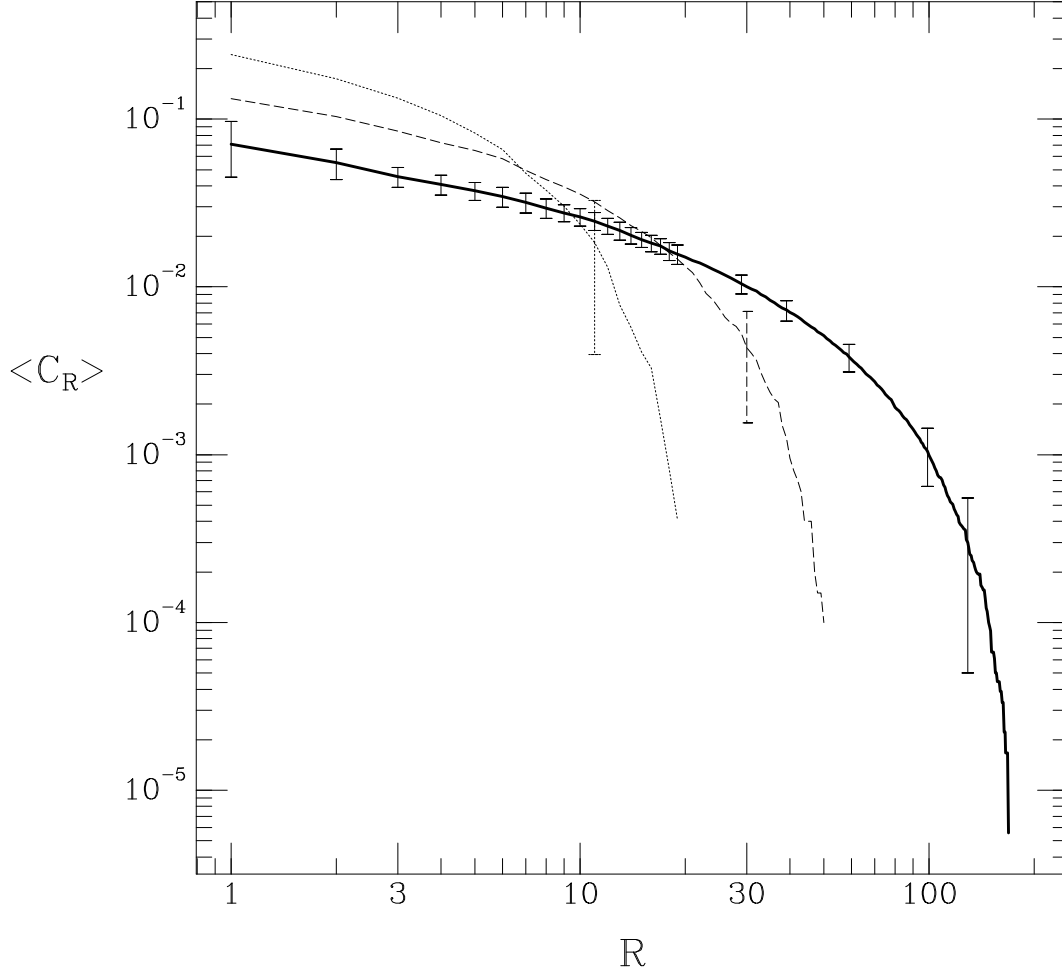


Fig. 4. Average fraction of cultural vectors $\langle C_R \rangle$ as a function of the rank R , for different system sizes with fixed parameter values $F_d = 1$, $F = 11$ and $q = 10 < q_c$. Continuous, dashed and dotted lines correspond to $N = 60 \times 60$, 20×20 and 7×7 , respectively. Error bars corresponding to one standard deviation are shown on several points.

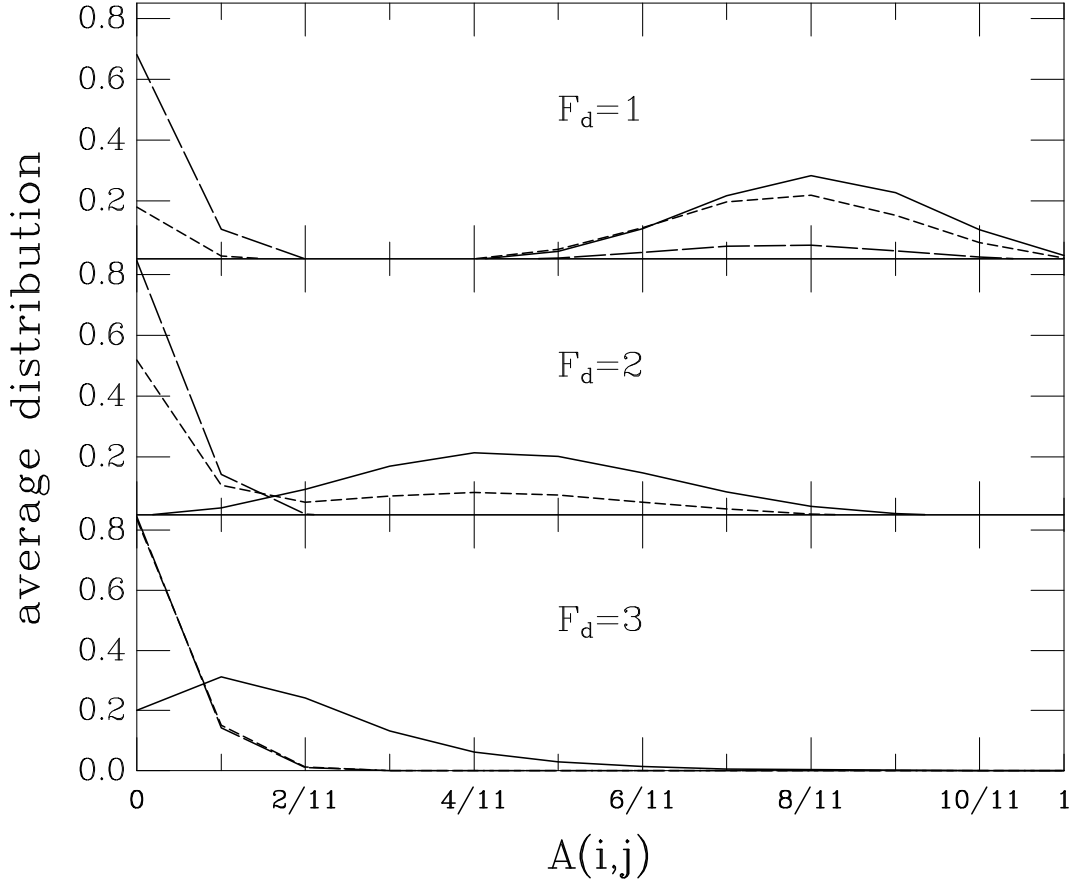


Fig. 5. Average distribution of the cultural affinities $A(i, j)$ of all pairs of elements in the system, averaged over 10 realizations, for the value of F_d indicated on each panel. The continuous, the dashed, and the long-dashed lines on each panel correspond to the values of $q = 10$, $q = 62$, and $q = 66$, respectively. System size is $N = 80 \times 80$, and $F = 11$.

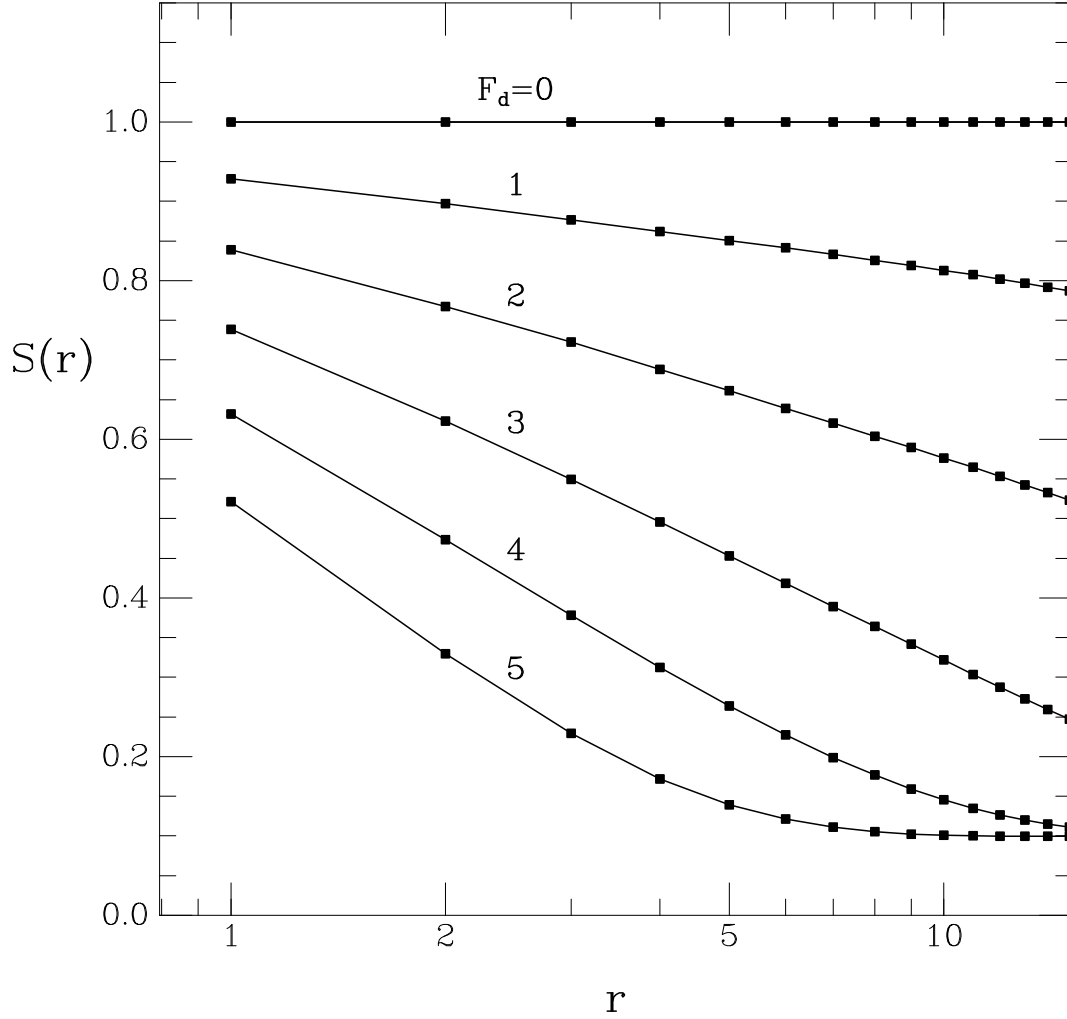


Fig. 6. The average shell affinity $S(r)$ for different values of F_d as indicated by the label besides each curve. System size is $N = 80 \times 80$, $F = 11$, and $q = 10$. The slope of each curve, $\alpha = -\frac{dS(r)}{d\log(r)}$, characterizes the *cultural cline*.