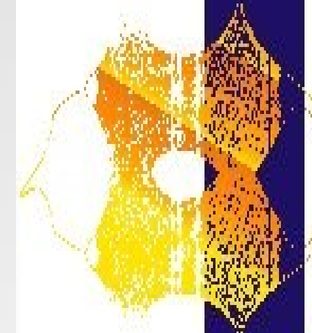
Universidad de Los Andes
Facultad de Ciencias

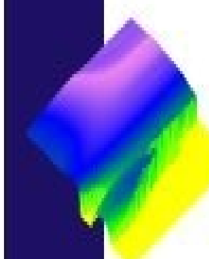
Defensa de Tesis de Licenciatura:

Comportamientos colectivos inducidos por diversidad en redes dinámicas



Br. Andrea Valdéz

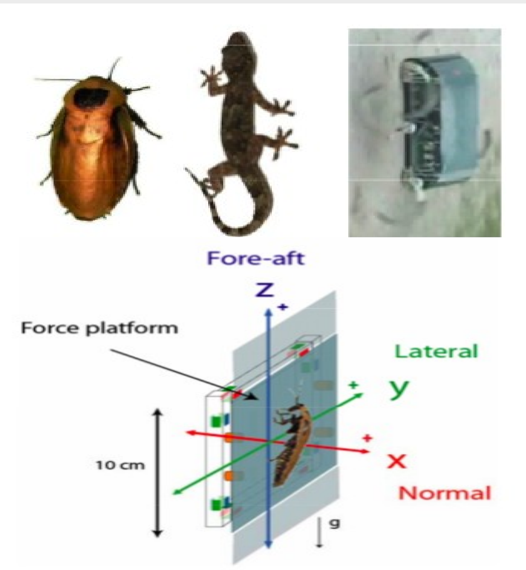
Tutor: Dr. Mario Cosenza

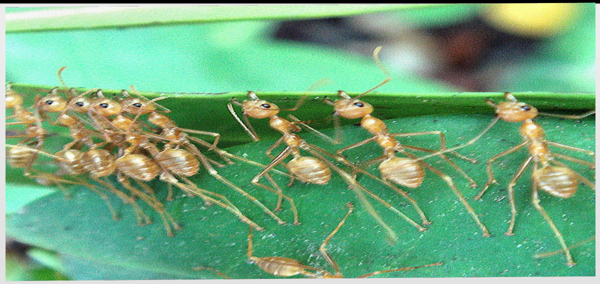


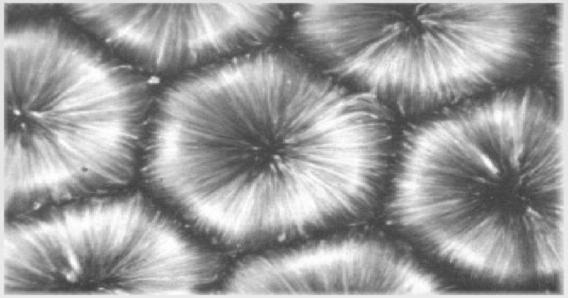
Centro de Física Fundamental
Área de Caos y Sistemas Complejos

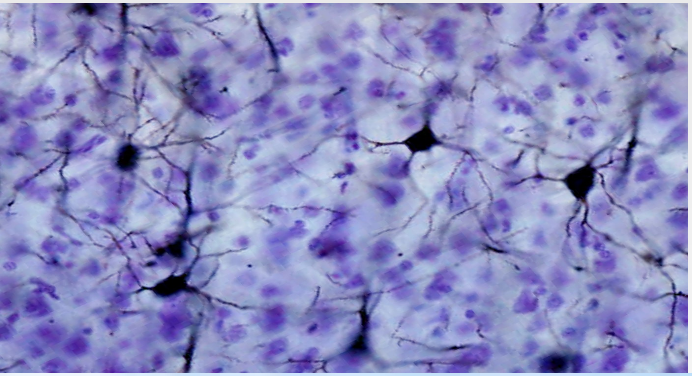
www.ciens.ula.ve/cff/caoticos

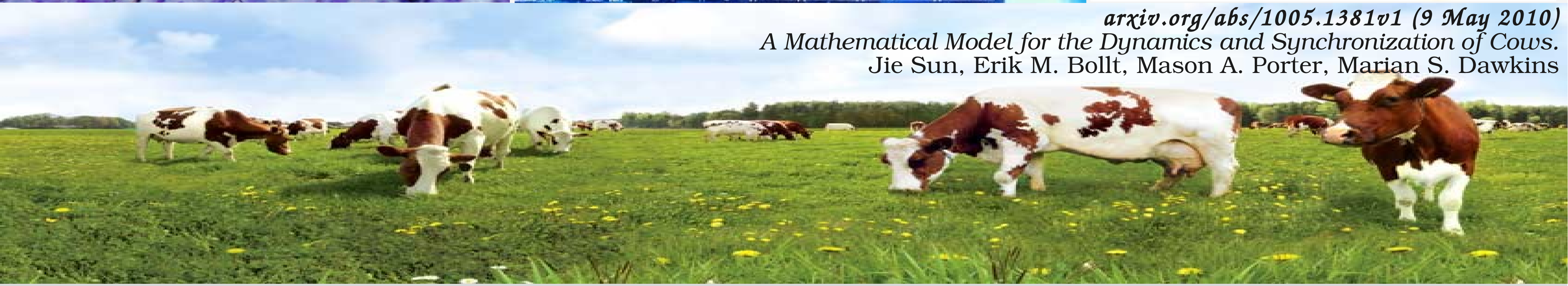
Comportamientos colectivos











arxiv.org/abs/1005.1381v1 (9 May 2010)
A Mathematical Model for the Dynamics and Synchronization of Cows.
Jie Sun, Erik M. Bollt, Mason A. Porter, Marian S. Dawkins

Caos robusto
Escenarios

- Mapas uniformes a trozos en 1-D

A. Potapov and M. K. Ali, Phy. Lett. A, 2000, 277(6): 310

$$x_{k+1}=\left|\tanh s\,(x_k-c)\right|$$

- Mapas unimodales uniformes en 1-D

M. Andrecut and M. K. Ali, Inter. J. Mod. Phys. B, 2001, 15(2): 177
M. Andrecut and M. K. Ali, Mod. Phys. Lett. B, 2001, 15(12-13): 391

$$f(x,\alpha)=\frac{1-x^{\alpha}-(1-x)^{\alpha}}{1-2^{1-\alpha}} \qquad f(\phi(x),\nu)=\frac{1-\nu^{\pm\phi(x)}}{1-\nu^{\pm\phi(c)}}$$

- Mapas uniformes a trozos en 2-D

Z. Elhadj and J. C. Sprott., *A Unified Piecewise Smooth Chaotic Mapping that conteins the Hénon and the Lozi Systems, submitted*

$$U(x,y)=\begin{pmatrix} 1-1.4 f_{\alpha}(x)+y \\ 0.3x \end{pmatrix} \qquad f_{\alpha}(x)=\alpha|x|+(1-\alpha)x^2$$

$$0\leqslant\alpha\leqslant 1$$

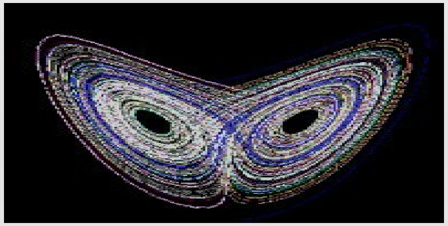
- Mapas no- uniformes

A. Priel and I. Kanter, Europhys. Lett., 2000, 51(2): 230

$$f(x)=\begin{cases} 1, & if \quad w.x+b>0 \\ 0, & else \end{cases}$$

- Sistemas tipo-Lorenz

E. N. Lorenz, J. Atmos. Sci., 1963, 20:130



$$\begin{cases} \dot{x}=\sigma(y-x) \\ \dot{y}=rx-y-xz \\ \dot{z}=-bz+xy \end{cases}$$

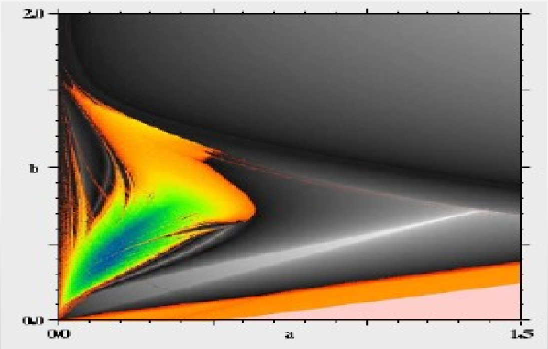
- Sistemas hiperbólicos

S. Kuznetsov and E. Seleznev, Journal of Experimental and Theoretical Physics, 2006, 102(2): 355

$$\begin{cases} \dot{x}=-2\pi u+(h_1+A_1\cos(2\pi\tau)/N)x-\frac{1}{3}x^3 \\ \dot{u}=2\pi(x+\epsilon_2 y\cos(2\pi\tau)) \\ \dot{y}=-4\pi v+(h_2-A_2\cos(2\pi\tau)/N)y-\frac{1}{3}y^3 \\ \dot{v}=4\pi(y+\epsilon_1 x^2) \end{cases}$$

- Modelo de Láseres

Gallas J., Int. J. of Bifurcation and Chaos, Vol. 20, No. 2 (2010) 197-211.



$$\begin{cases} \dot{x}=y \\ \dot{y}=x(1-z)-Bx^3-by \\ \dot{z}=-a(z-x^2) \end{cases}$$

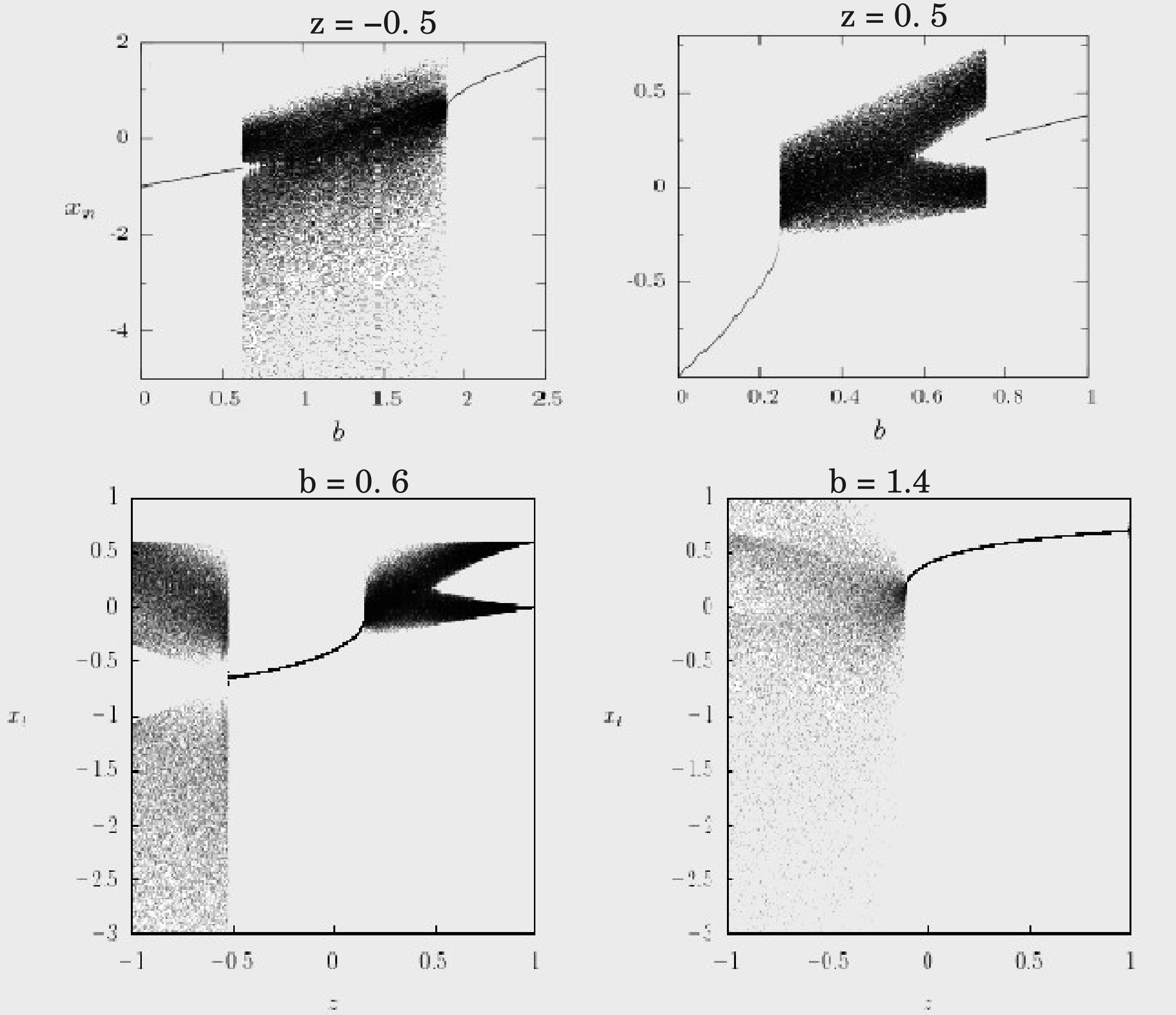
$$B=0$$

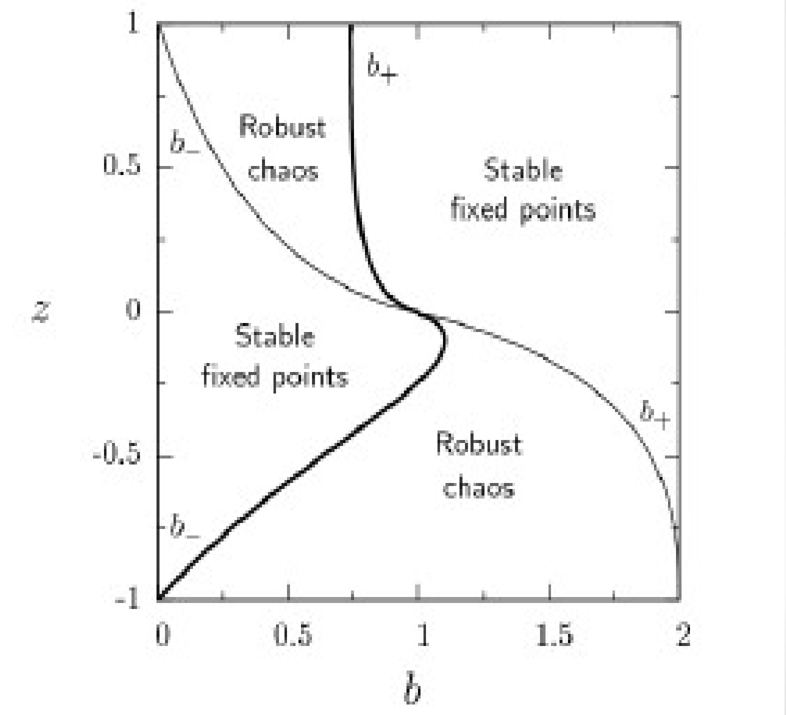
Escenario de caos robusto

Mapas Singulares

- $f(x_t) = b - |x_t|^z$

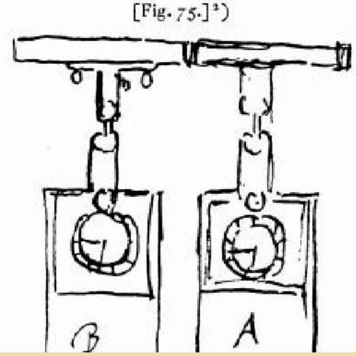
$|z| < 1$





O. Alvarez and M. G. Cosenza. CIENCIA 15(4), 438 - 443, 2007

⭐ *Comportamientos colectivos en RMAG:* ***Sincronización***



huygens' clocks

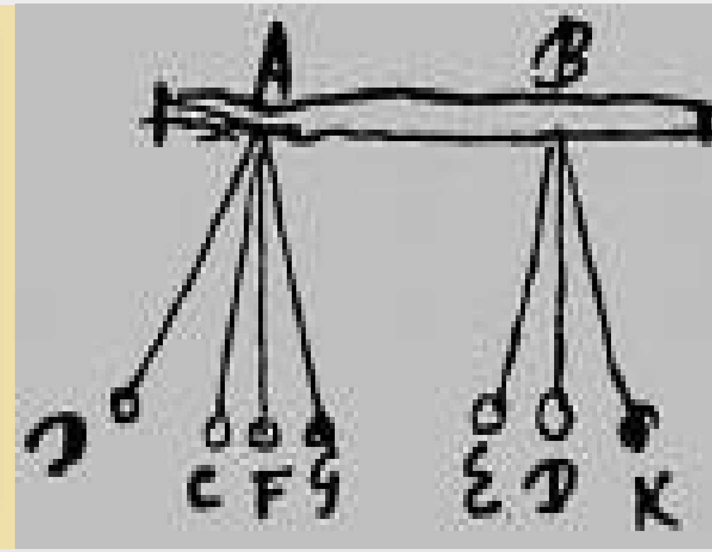
V. [1]

1665.

[Fig. 75.] [2]

22 febr. 1665.

Diebus 4 aut 5 horologiorum duorum novorum in quibus catenulæ [Fig. 75], miram concordiam obſervaveram, ita ut ne minimo quidem exceſſu alterum ab altero ſuperaretur. ſed conſonarent ſemper reciprocationes utriusque perpendiculi. unde cum parvo ſpatio inter ſe horologia diſtarent, ſympathiæ quandam [3]) quasi alterum ab altero afficeretur ſuſpicari cœpi. ut experimentum caperem turbavi alterius penduli reditus ne ſimul incederent ſed quadrante horæ poſt vel ſemihora rurſus concordare inveni.



$$\langle \sigma \rangle = \frac{1}{T - t_0} \sum_{t=t_o}^{T} \sigma_t$$

$$\sigma(t) = \sqrt{\langle x_t^2 \rangle - \langle x_t \rangle^2}$$

$$\langle x_t^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} x_t^2(i)$$

$$\langle x_t \rangle = \frac{1}{N} \sum_{j=1}^{N} x_t(j)$$

- Condición para la sincronización: $\langle \sigma \rangle = 0$

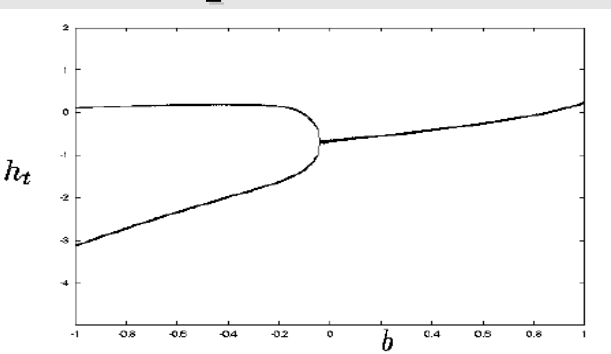
$$\rightarrow \; x_t(i) = x_t(j), \quad \forall\, i, j$$

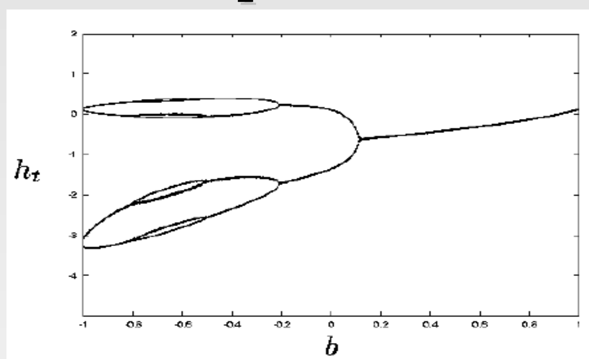
Comportamientos colectivos en RMAG: **Estados desincronizados**

Redes de mapas globalmente acoplados (RMAG)[1]

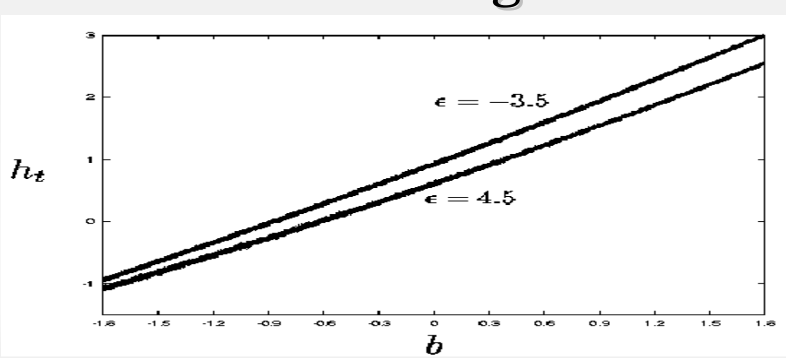
$$x_{t+1}(i) = (1-\epsilon) f(x_t(i)) + \frac{\epsilon}{N}\sum_{j=1}^{N} f(x_t(j)) \quad (i = 1, 2, 3, \ldots, N) \longrightarrow \quad h_t = \frac{1}{N}\sum_{j=1}^{N} f(x_t(j))$$

- Comportamientos colectivos periódicos[2]

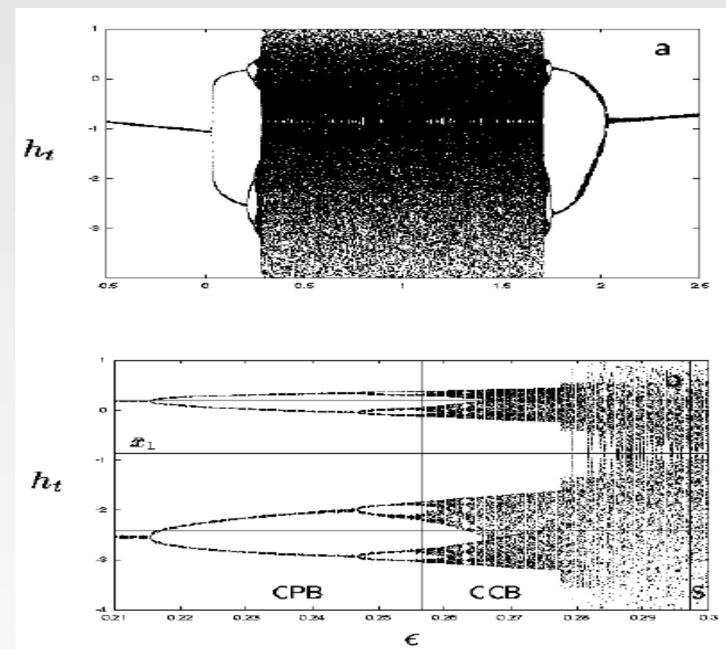




- Turbulencia global[2]



- Bandas caóticas[2]



➡ Diagramas de bifurcación del campo medio de mapas logarítmicos: $f(x_t) = b + \ln|x_t|$ globalmente acoplados en función de los parámetros b y ϵ. En el caso homogéneo.

[1] K. Kaneko, Phys. Rev. Lett., 63, 219 (1989)

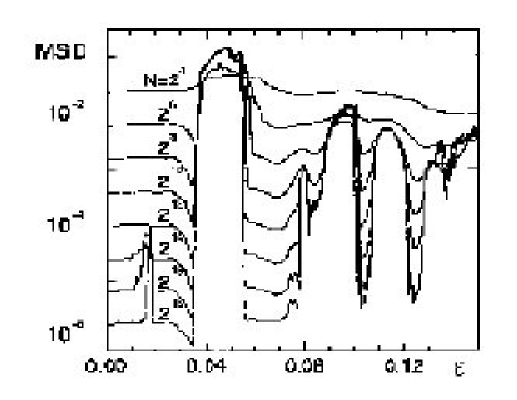
[2] M. G. Cosenza and J. González; Prog. Theor. Phys. 100, 21 (1998)

Antecedentes de la heterogeneidad en RMAG: ***Heterogeneidad con mapas logísticos***

- Distribución del parámetro local *a* :

$$f_i(x) = 1 - a(i)x^2$$

$$a(i) = a_0 + \frac{\Delta a(2i - N)}{2N} \longrightarrow \left[a_0 - \frac{\Delta a}{2}, a_0 + \frac{\Delta a}{2}\right]$$



$$\langle(\delta h)^2\rangle = \langle(h - \langle h\rangle)^2\rangle$$

"La DCM se mantiene finita para N grande"

- Ruido interno :

$$f^i(y) = 1 - ay^2 + s(i)$$

"El ruido induce la sincronización"

- Distribución de los parámetros de acoplamiento ϵ :

$$x_{t+1}(i) = (1 - \epsilon(i))f(x_t(i)) + \frac{\epsilon(i)}{N}\sum_{j=1}^{N} f(x_t(j))$$

$$\epsilon(i) = \epsilon_{min} + (\epsilon_{max} - \epsilon_{min})(i/N).$$

"La formación de clusters depende de ϵ"

- Ruido externo :

$$x_{t+1}(i) = (1 - \epsilon)f_i(x_t(i)) + \frac{\epsilon}{N}\sum_{j=1}^{N} f_j(x_t(j)) + \xi_t(i).$$

"El ruido microscópico puede suprimir los grados de libertad en un nivel macroscópico"

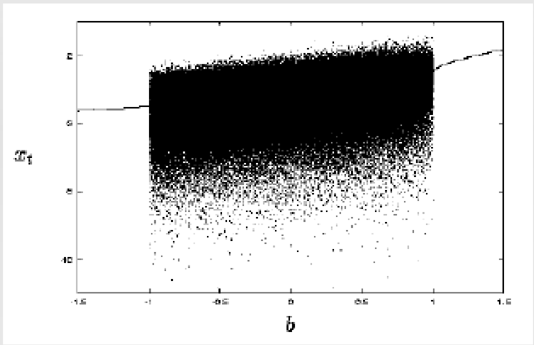
T. Shibata and K. Kaneko, Europhys. Lett., 38 (6), pp. 417-422 (1997)

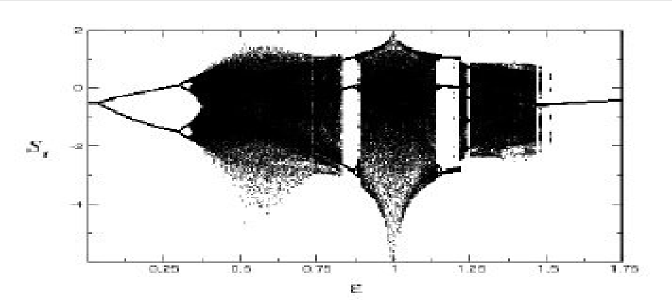
K. Kaneko; Physica D 55, 368-384 (1992)

Antecedentes de la heterogeneidad en RMAG: **Heterogeneidad con mapas logarítmicos**

- Distribución del parámetro local b :

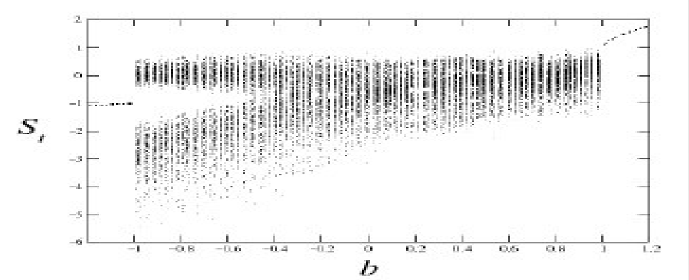
$$f_i(x_t) = b(i) + \ln|x_t|$$





- Distribución del parámetro de acoplamiento **ϵ** :

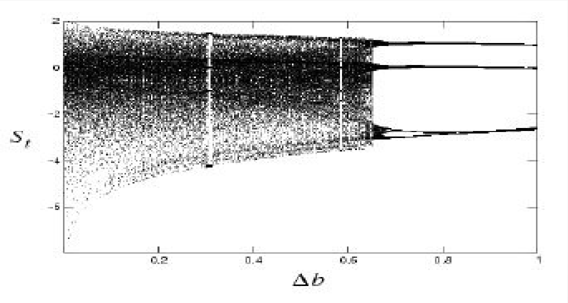
$$x_{t+1}(i) = (1-\epsilon(i))f(x_t(i)) + \frac{\epsilon(i)}{N}\sum_{j=1}^{N} f(x_t(j))$$



- Análisis del campo medio en función de Δb:

$$b(i) \in [-\Delta b, \Delta b]$$

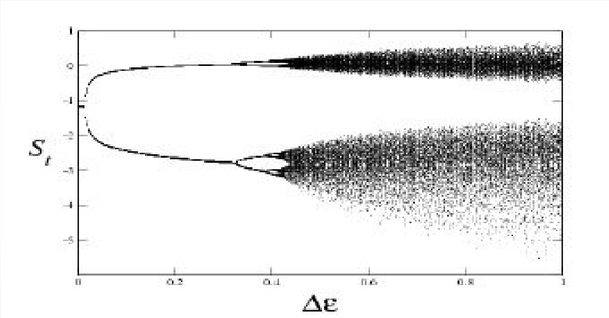
$$(\Delta b = b_0 + h)$$



- Análisis del campo medio en función de Δ**ϵ**:

$$\epsilon(i) \in [-\Delta\epsilon, \Delta\epsilon]$$

$$(\Delta\epsilon = \epsilon_0 + h)$$

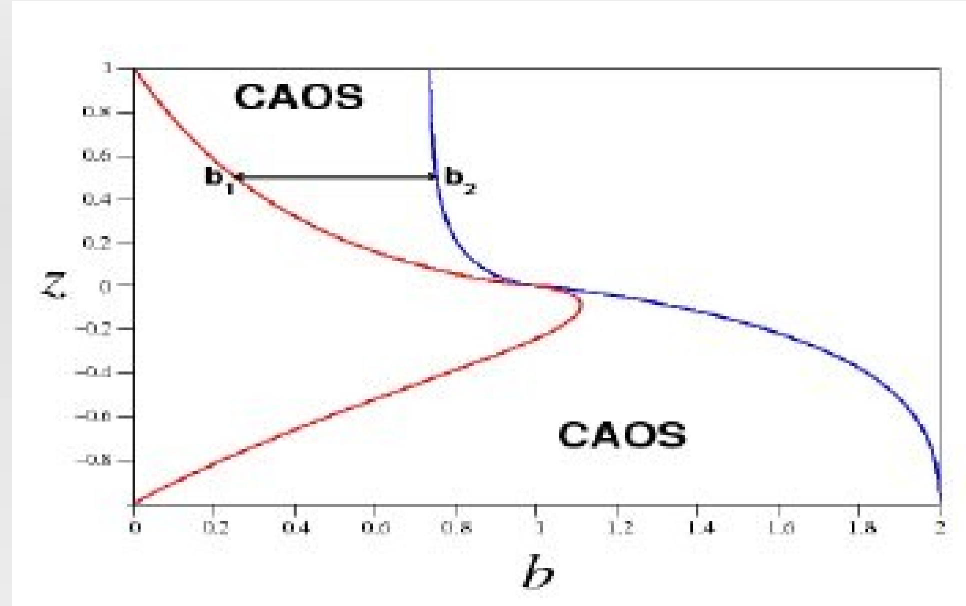


Javier González Estévez, Tesis de Maestría, Universidad de Los Andes, Mérida (2001)

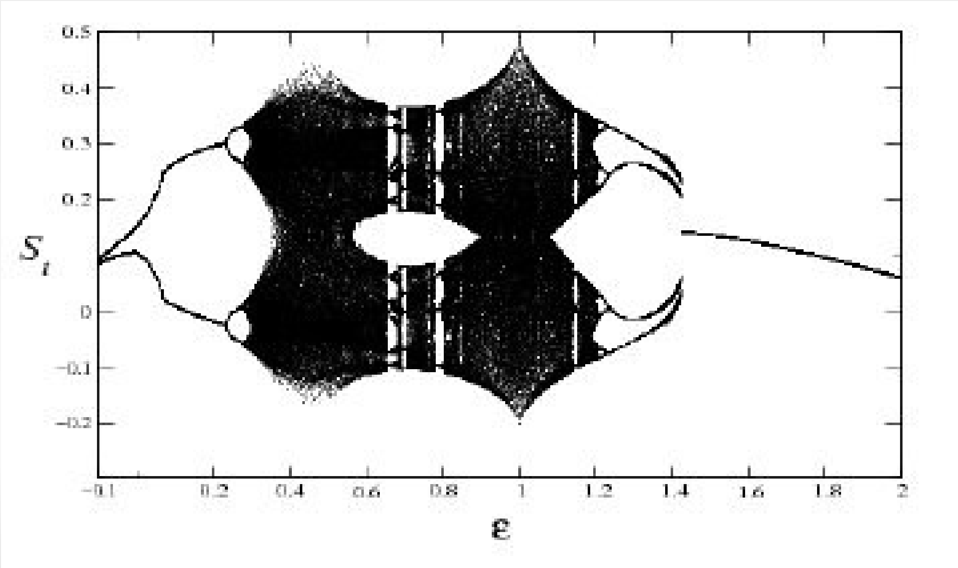
Antecedentes de la heterogeneidad en RMAG: ***Heterogeneidad con mapas singulares***

- Distribución del parámetro local b :

$$f_i(x_t(i)) = b(i) - |x_t|^z$$



$$x_{t+1}(i) = (1-\epsilon)f(x_t(i)) + \frac{\epsilon}{N}\sum_{j=1}^{N} f(x_t(j)) \quad (i = 1, 2, 3, ..., N)$$



→ *"cangrejo"*

Javier González Estévez, Tesis de Maestría, Universidad de Los Andes, Mérida (2001)

⭑ ***Heterogeneidad en los acoplamientos*** *de una RMAG de mapas singulares*

- Nuestro modelo con acoplamiento heterogéneo

$$x_{t+1}(i) = (1-\epsilon(i))f(x_t(i)) + \frac{\epsilon(i)}{N}\sum_{j=1}^{N} f(x_t(j))$$

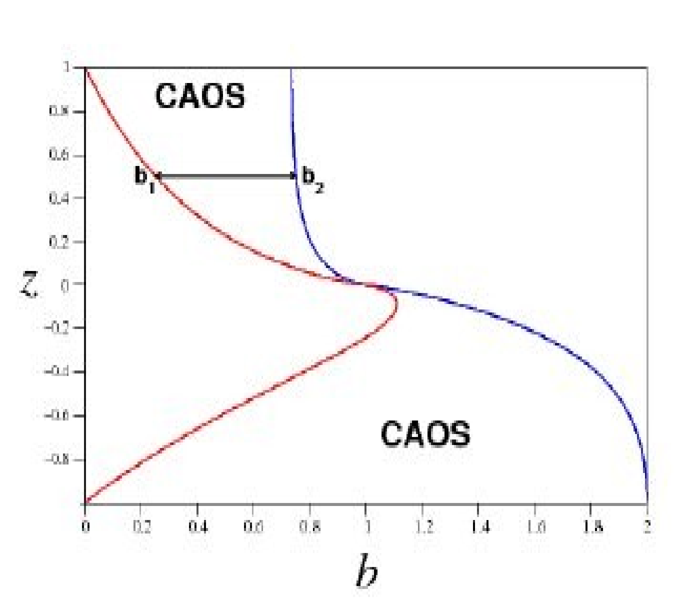
$$f(x_t) = b - |x_t|^z$$

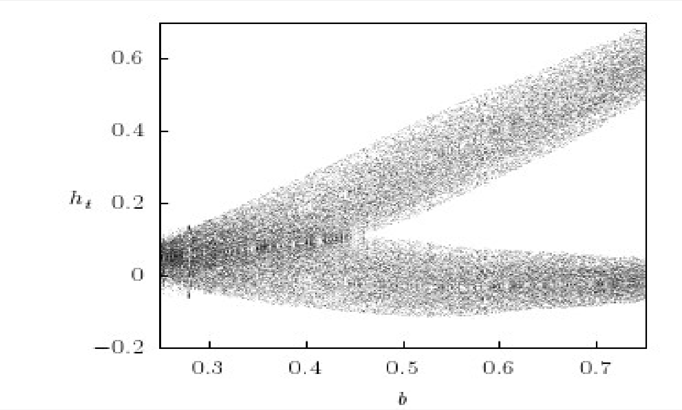
⭑ Comportamientos colectivos en función de *b*

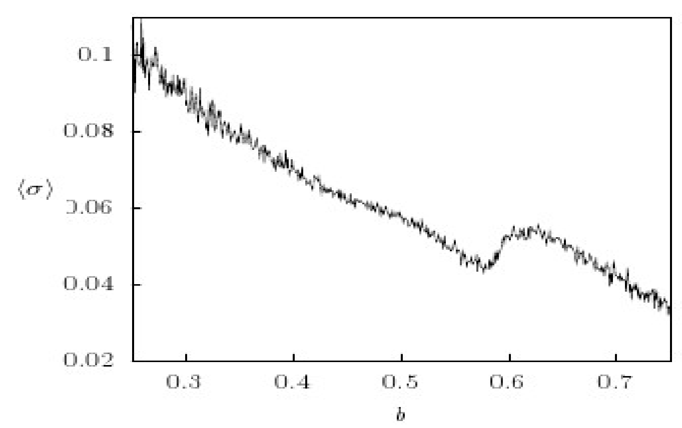
$$\boxed{\epsilon(i) \in [0,2]}$$

$$z = 0.5$$

$$[b_1 = 0.250, b_2 = 0.750]$$





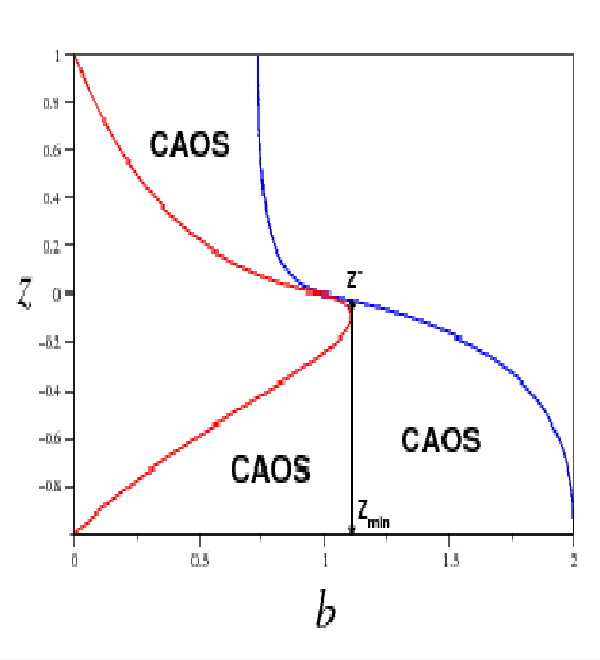


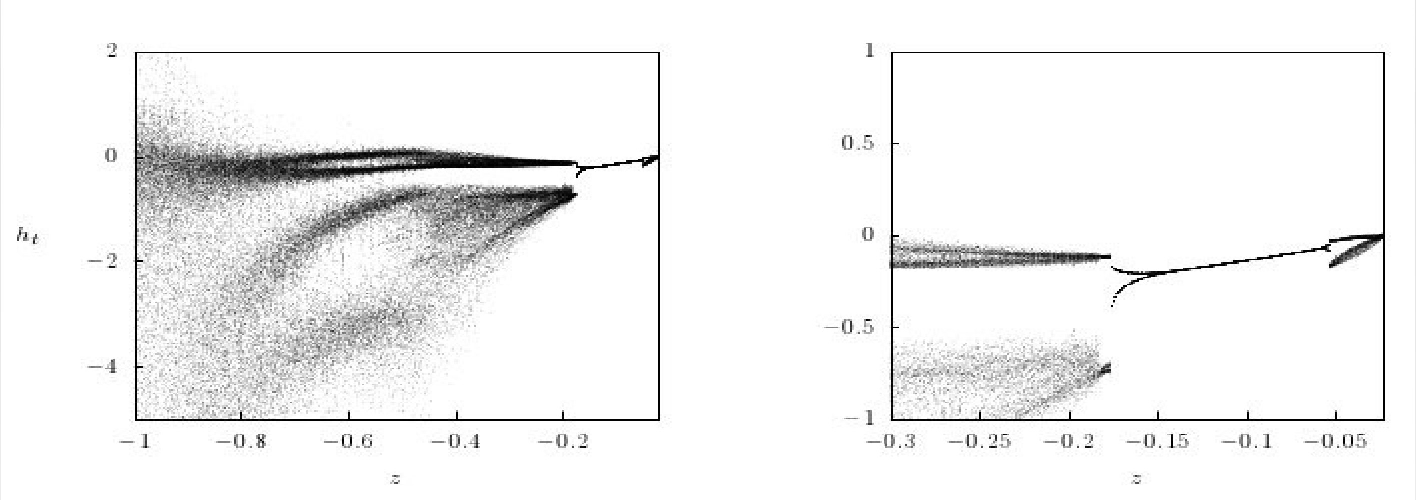
⭑ ***Heterogeneidad en los acoplamientos*** *de una RMAG de mapas singulares*

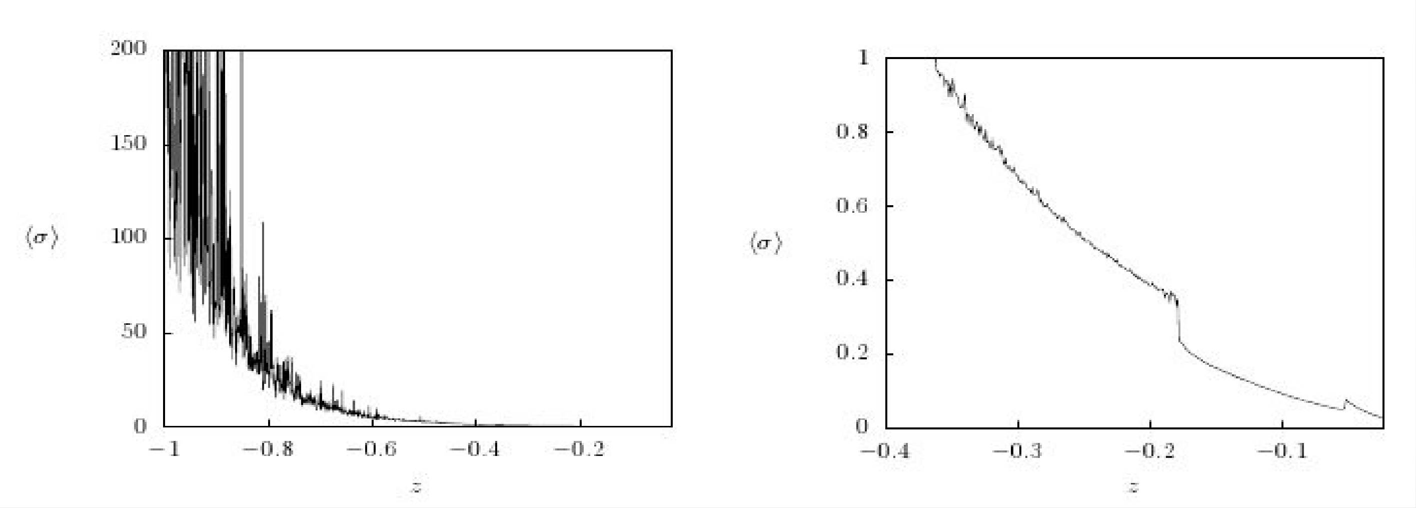
⭑ Comportamientos colectivos en función de *z*

$$\boxed{\epsilon(i) \in [-0.5, 2.5]}$$

$b = 1.11$ $z \in [-1, -0.023]$



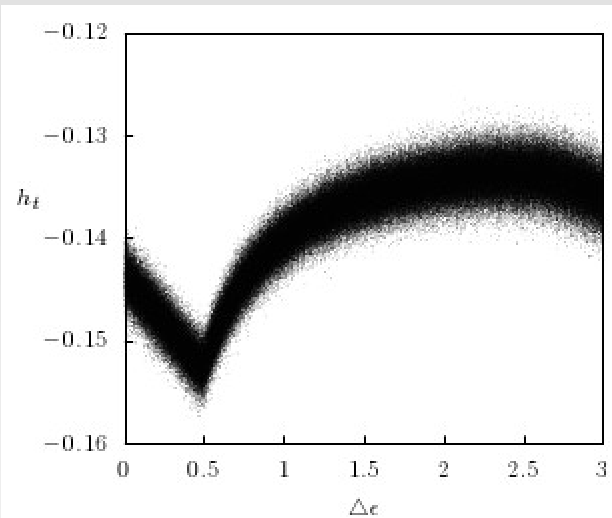


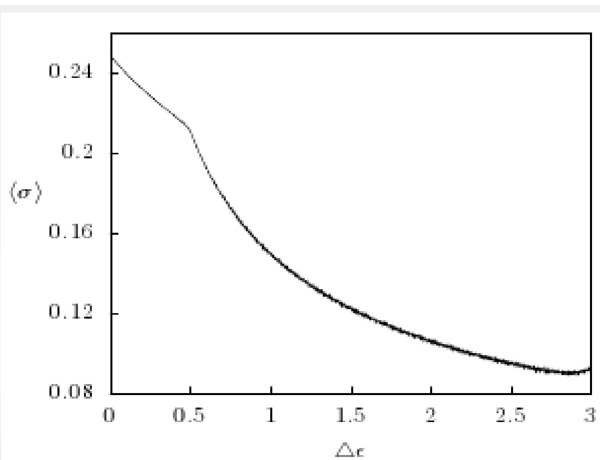


Influencia de la heterogeneidad en los acoplamientos de una RMAG de mapas singulares

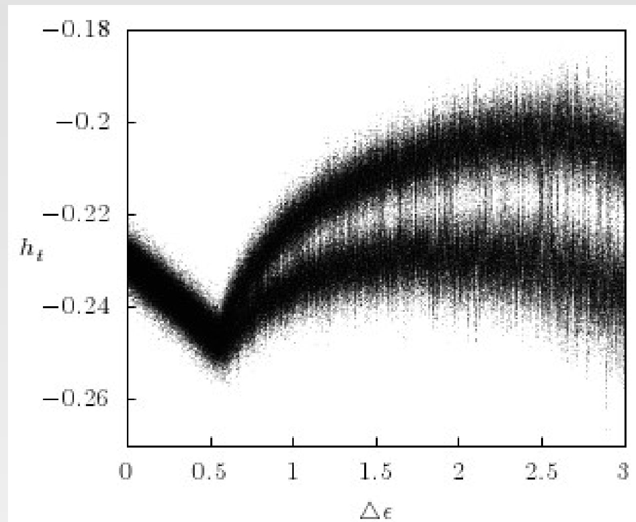
$$\epsilon(i) = \epsilon_{min} + \triangle\epsilon \cdot \gamma \qquad (\triangle\epsilon \in [0, \epsilon_{max} - \epsilon_{min}])$$

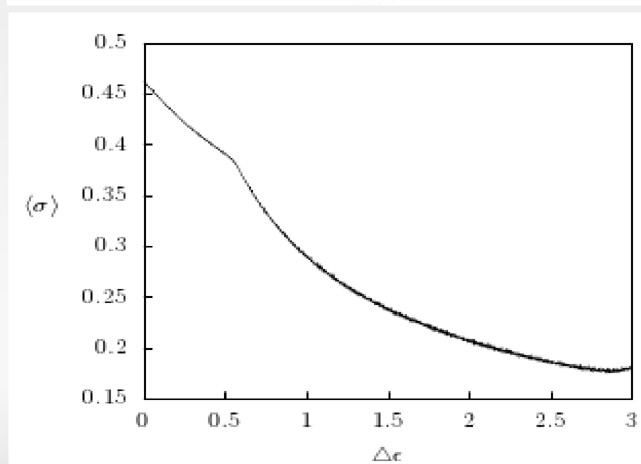
Con $b = 1.11$ y $z = -0.10$





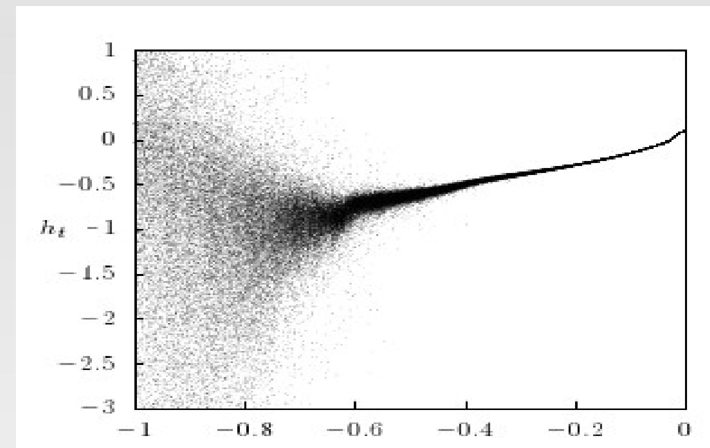
Con $b = 1.11$ y $z = -0.16$



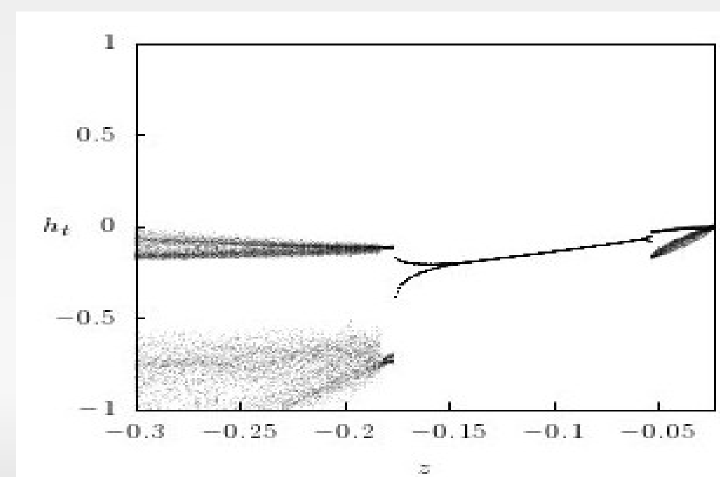


$[\epsilon_{min} = -0.5, \ \ \epsilon_{max} = 2.5]$

Campo medio h_t en función del exponente z.



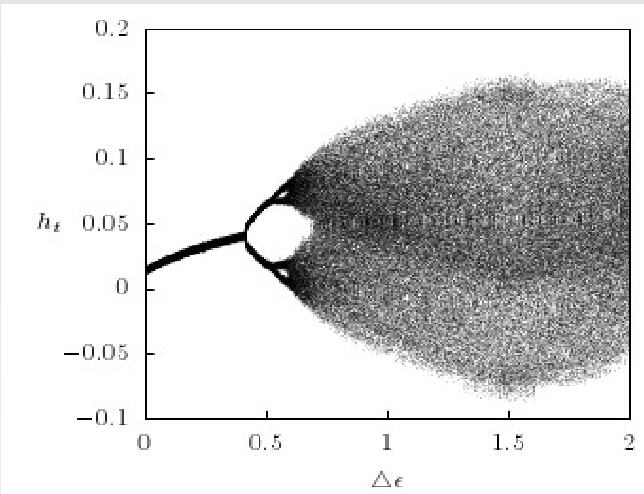
$b = 1.11$ y $\epsilon = -0.5$

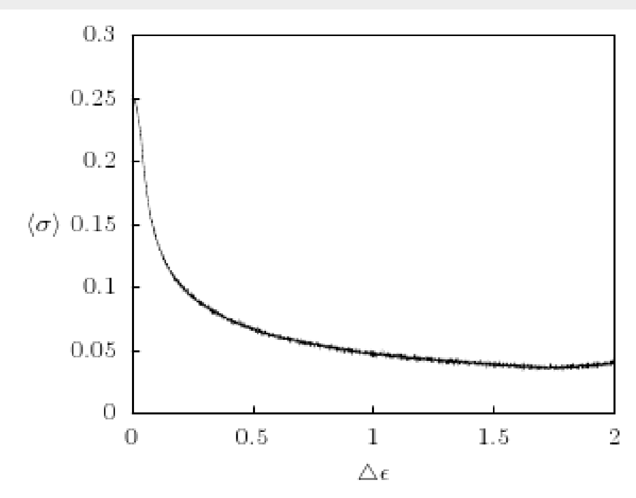


$b = 1.11$ $\quad \epsilon(i) \in [-0.5, 2.5]$

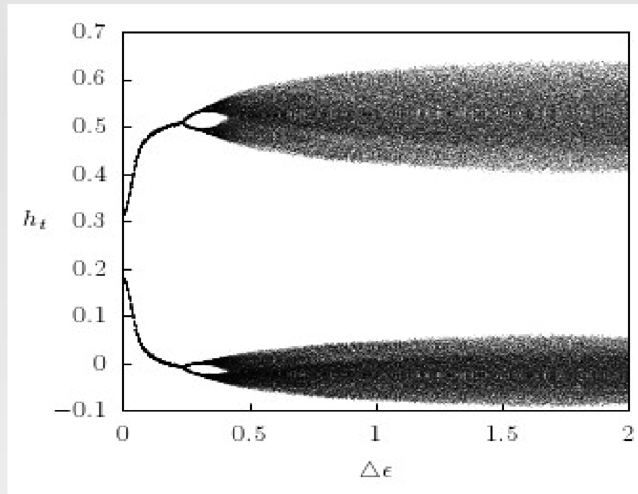
Influencia de la heterogeneidad en los acoplamientos de una RMAG de mapas singulares

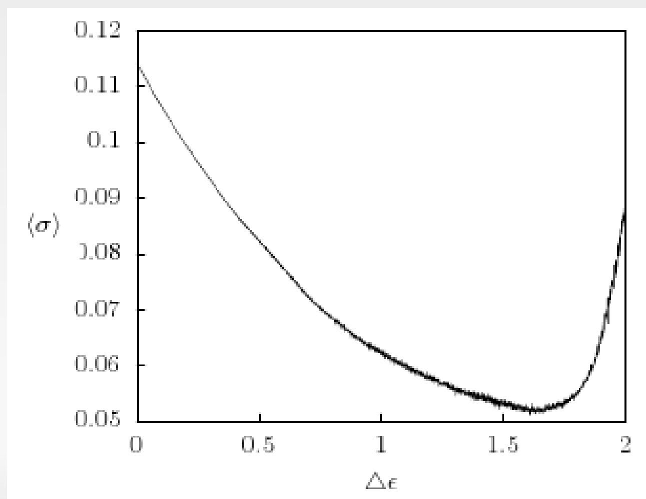
Con $b = 0.3$ y $z = 0.5$





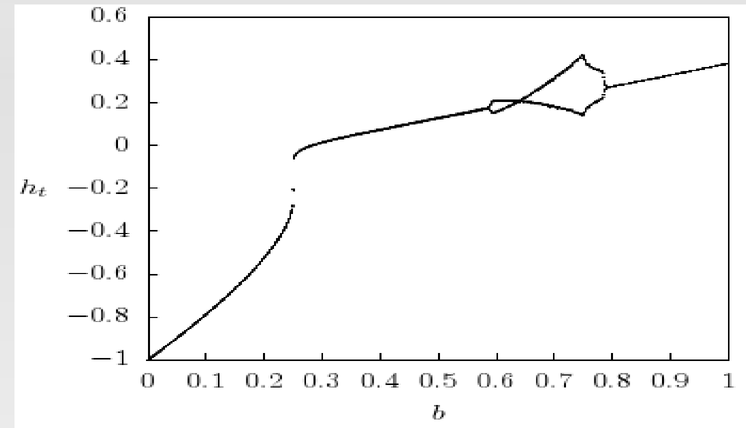
Con $b = 0.7$ y $z = 0.5$



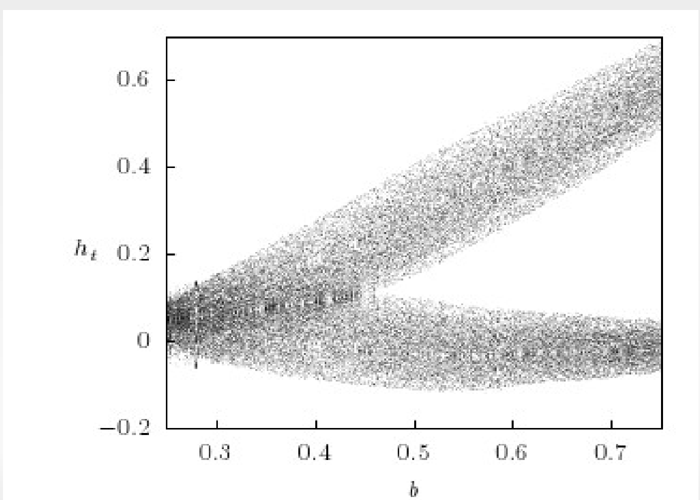


$[\epsilon_{min}=0,\ \epsilon_{max}=2]$

Campo medio h_t en función del parámetro local b



Con $z = 0.5$ y $\epsilon = 0$



$z = 0.5$ $\epsilon(i) \in [0, 2]$

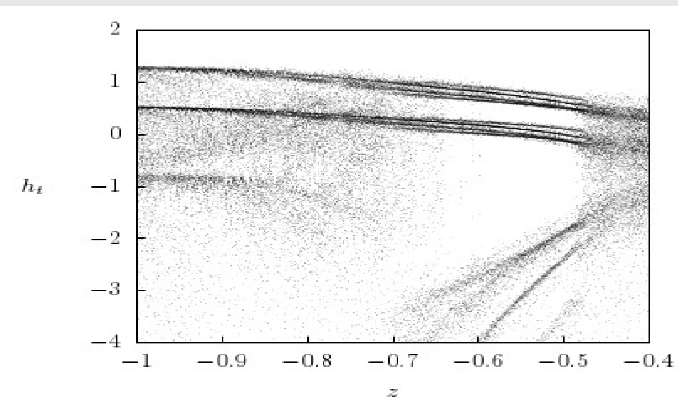
⋆ ***Heterogeneidad en el parámetro local b*** *de una RMAG de mapas singulares*

- Nuestro modelo con dinámica local heterogénea

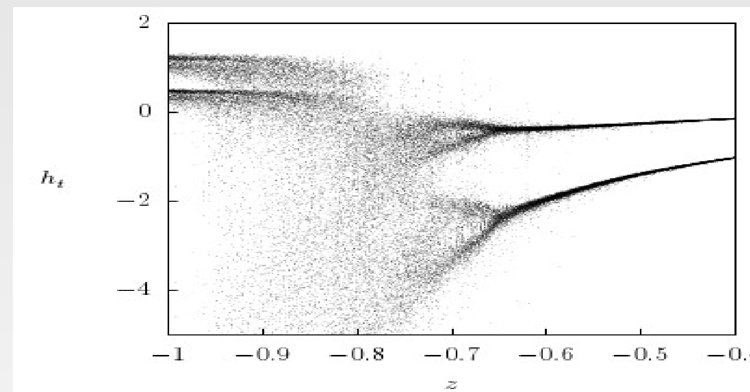
$$x_{t+1}(i) = (1-\epsilon) f_i(x_t(i)) + \frac{\epsilon}{N} \sum_{j=1}^{N} f_j(x_t(j)) \quad (i = 1, 2, 3, \ldots, N).$$

$$f_i(x_t(i)) = b(i) - |x_t|^z$$

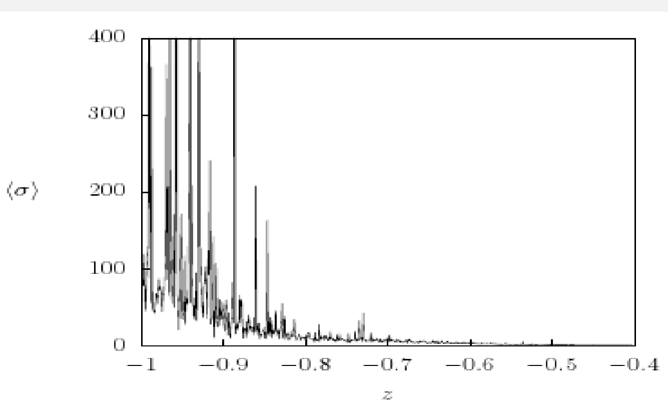
⋆ Comportamientos colectivos en función de *z*



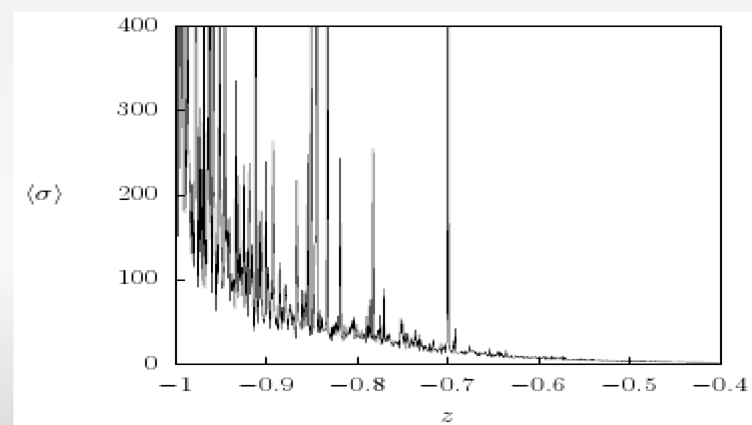
$\epsilon = 0.5$



$\epsilon = 0.2$



$\boxed{b(i) \in [0.875, 1.75]}$



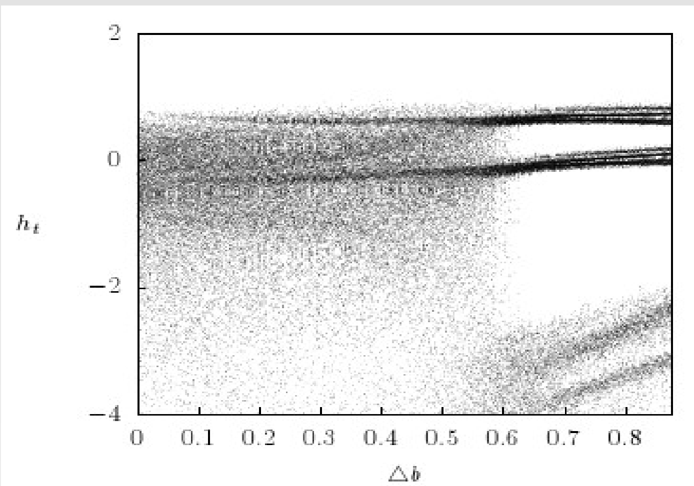
⭑ *Influencia de la heterogeneidad en los parámetros locales b de una RMAG de mapas singulares*

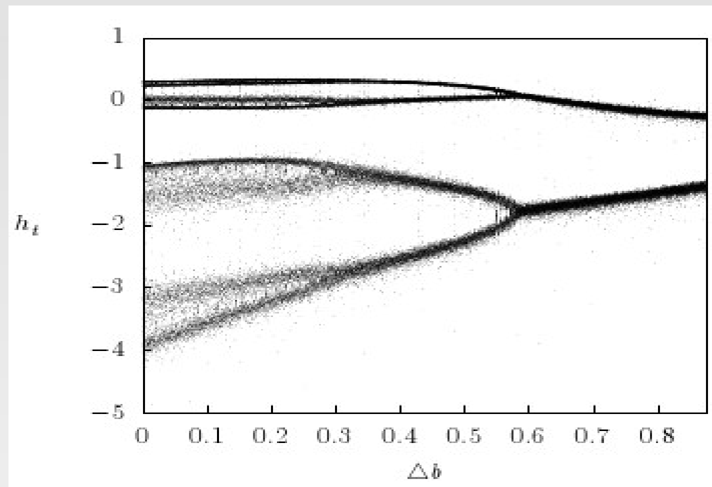
$$b(i) = b_{min} + \triangle b \cdot \gamma, \qquad \triangle b \in [0, b_{max} - b_{min}]$$

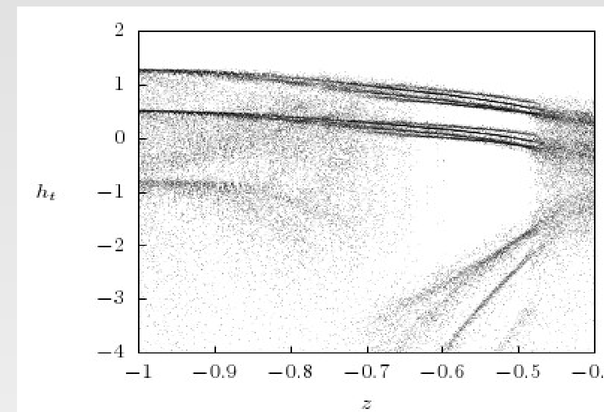
$\epsilon = 0.5$ y $z = -0.55$

$\epsilon = 0.2$ y $z = -0.5$

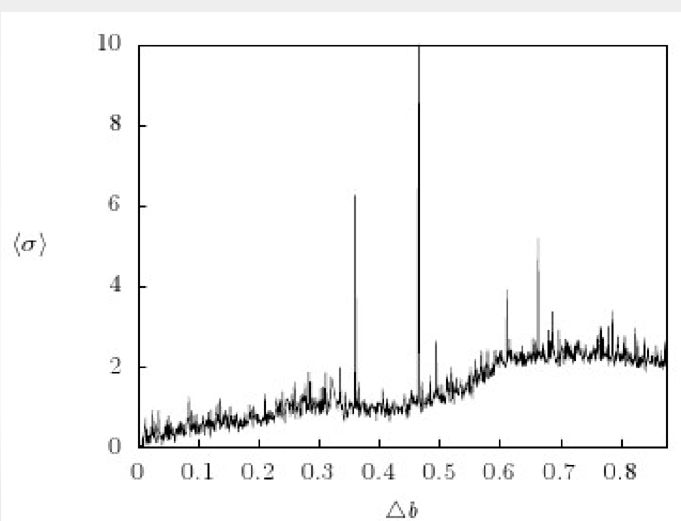
$b(i) \in [0.875, 1.75]$

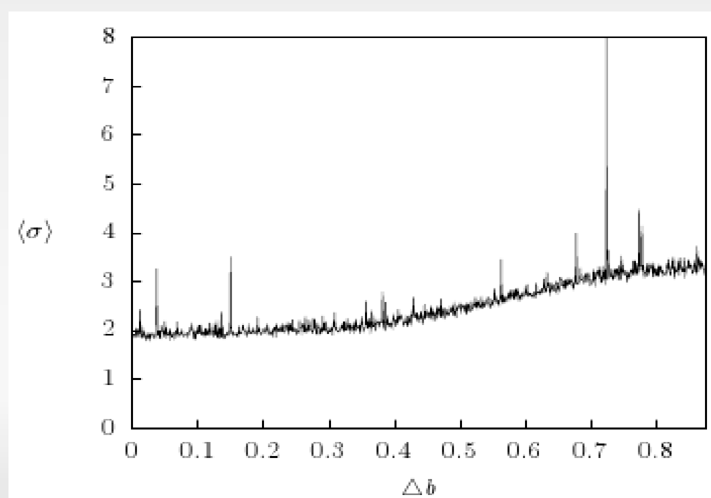


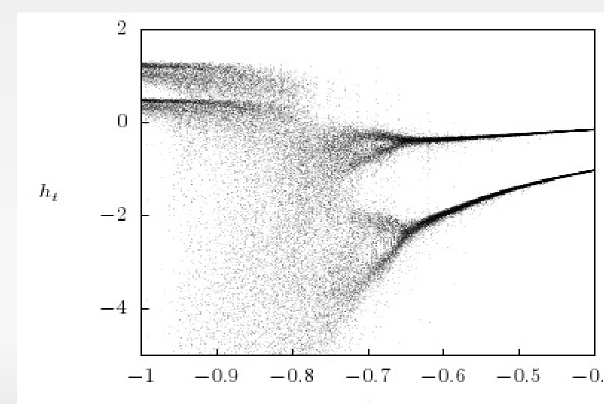




$\epsilon = 0.5$







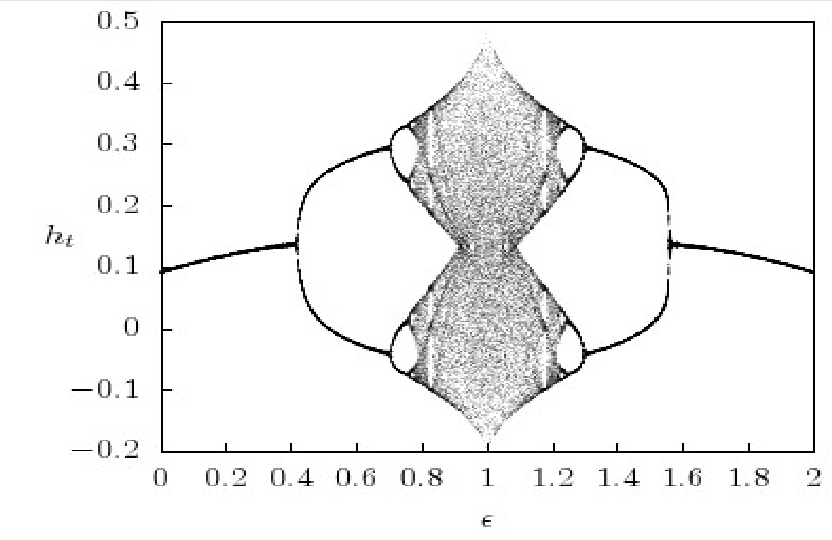
$\epsilon = 0.2$

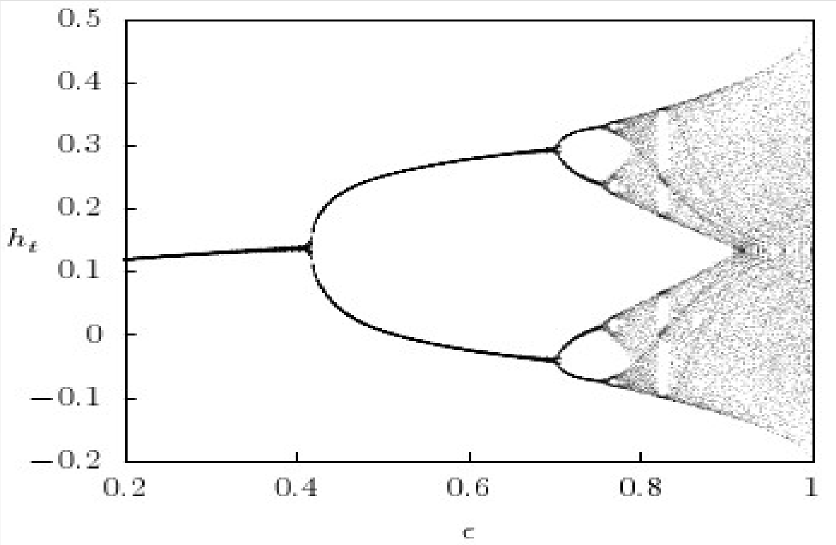
$[b_{min} = 0.875, \;\; b_{max} = 1.75]$

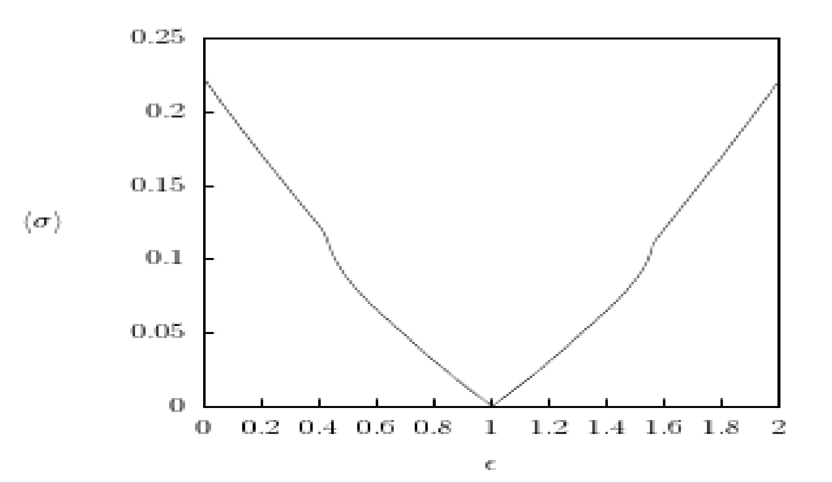
⋆ *Heterogeneidad variable en el parámetro local b de una RMAG de mapas singulares*

⋆ *Consideramos la dinámica local:* $f_i(x_t(i)) = b_t(i) - |x_t|^z$

$$x_{t+1}(i) = (1-\epsilon) f_i(x_t(i)) + \frac{\epsilon}{N}\sum_{j=1}^{N} f_j(x_t(j)) \quad (i = 1, 2, 3, \ldots, N).$$







$$\boxed{b_t(i) \in [0.250, 0.750]}$$

$$z = 0.5$$

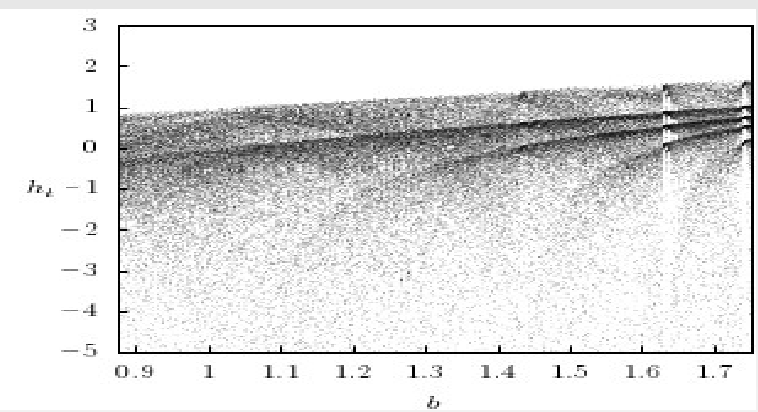
⋆ ***Heterogeneidad en el exponente z*** *de una RMAG de mapas singulares*

- Nuestro modelo con exponente de singularidad heterogéneo

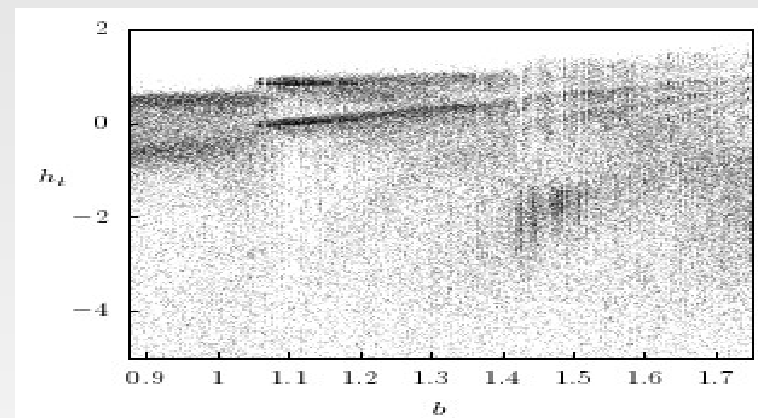
$$x_{t+1}(i) = (1-\epsilon) f_i(x_t(i)) + \frac{\epsilon}{N} \sum_{j=1}^{N} f_j(x_t(j)) \quad (i = 1, 2, 3, \ldots, N).$$

$$f_i(x_t(i)) = b - |x_t|^{z(i)}$$

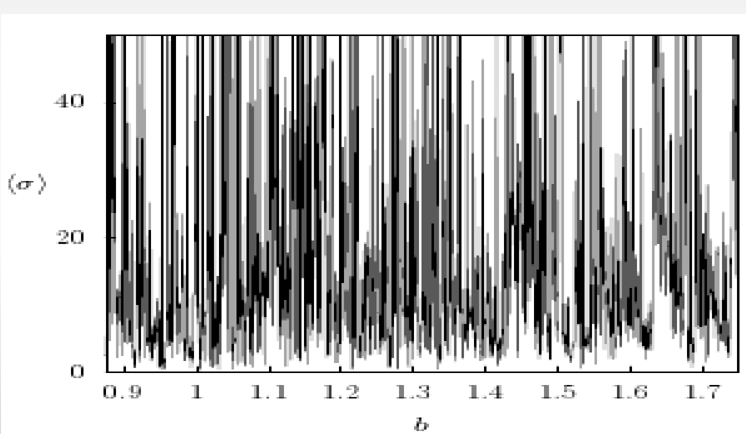
⋆ Comportamientos colectivos en función de *b*



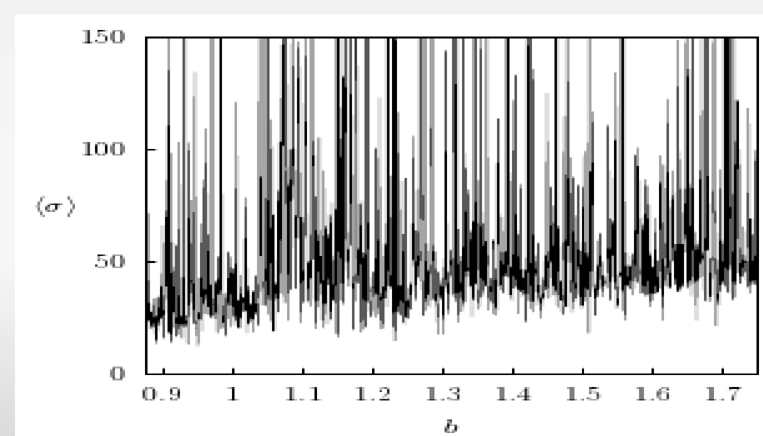
$\epsilon = 0.5$



$\epsilon = 0.2$



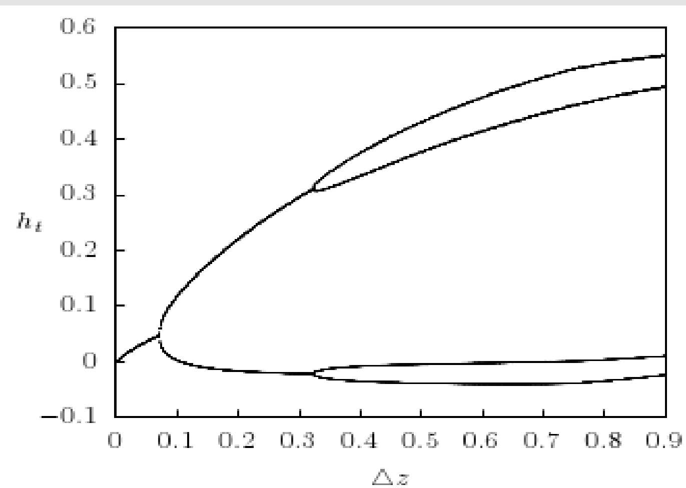
$z(i) \in [-1, -0.4]$

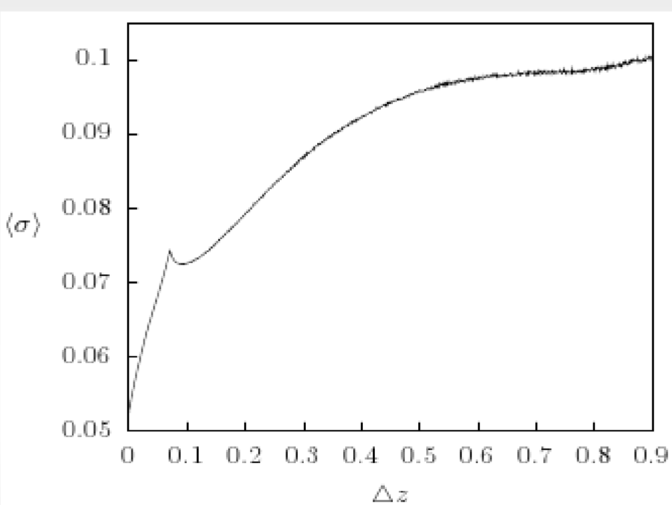


Influencia de la heterogeneidad en los exponentes z de una RMAG de mapas singulares

$$z(i) = z_{min} + \triangle z \cdot \gamma \qquad (\triangle z \in [0, z_{max} - z_{min}])$$

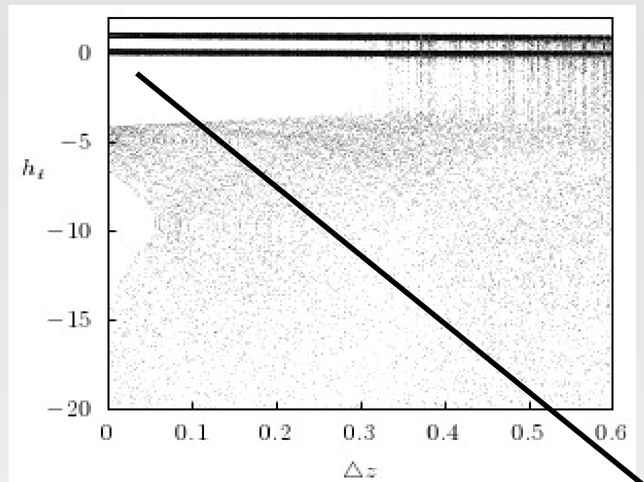
$b = 0.735$ y $\epsilon = 0.2$

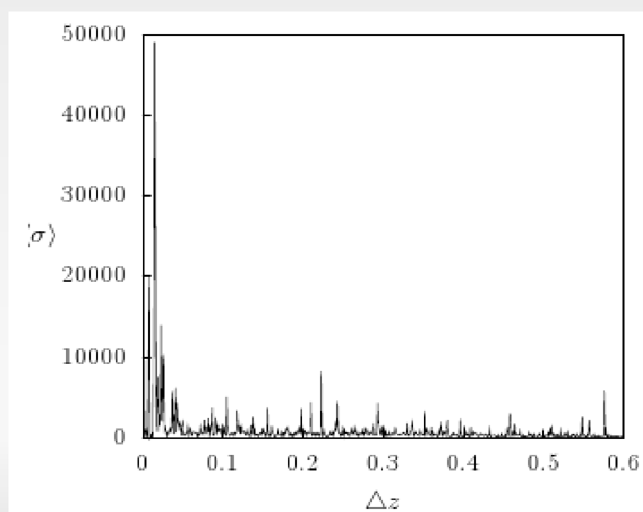




$[z_{min} = 0.85, \; z_{max} = 1]$

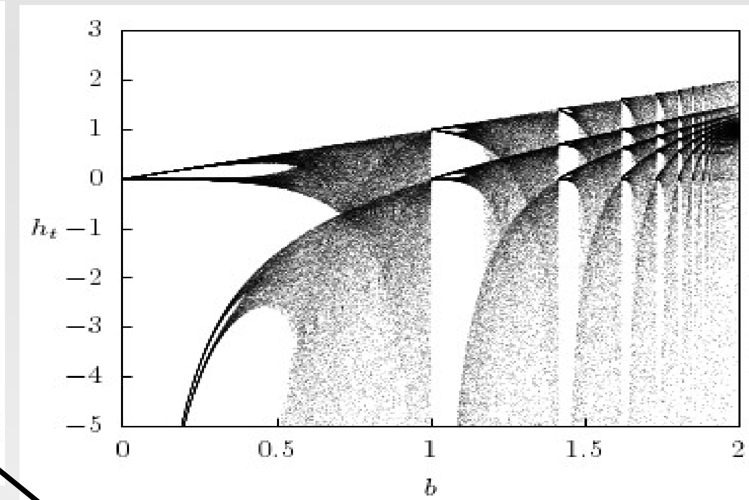
$b = 1.1$ y $\epsilon = 0.2$



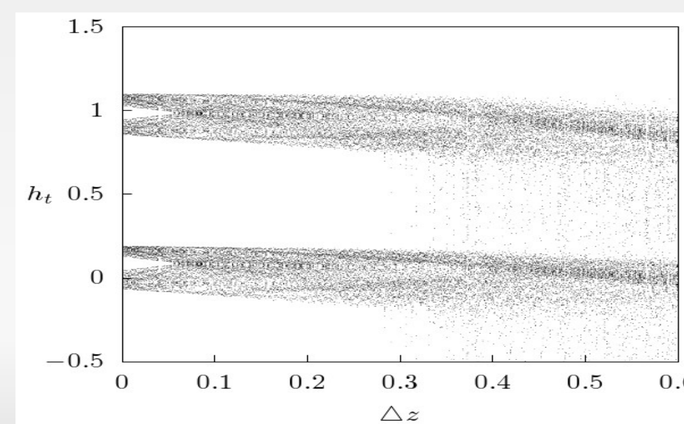


$[z_{min} = -1, \; z_{max} = -0.4]$

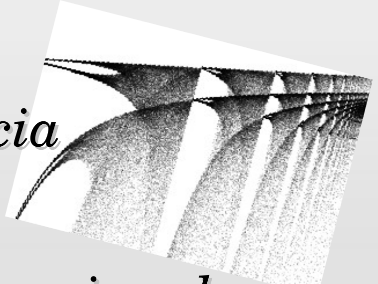
$\triangle z = 0$



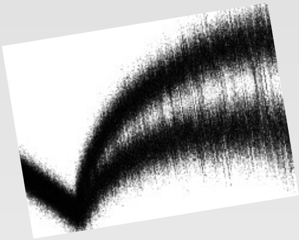
Con $z = -1$ y $\epsilon = 0.2$



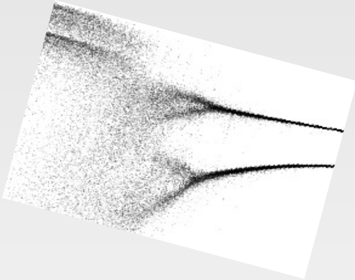
Conclusiones



- Los comportamientos colectivos que surgen en una RMAG, con dinámica local caótica, por lo general, son complejos, como consecuencia de las interacciones globales entre los elementos de la red.

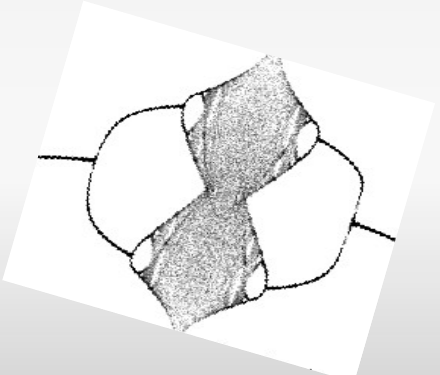


- La heterogeneidad en los acoplamientos entre los mapas singulares inducen comportamientos colectivos ordenados, además de favorecer la tendencia hacia la sincronización del sistema.



- La heterogeneidad en los parámetros locales b puede favorecer la sincronización en ciertos rangos de valores del exponente z, y sin duda a la emergencia de comportamientos colectivos no triviales.

- En ciertos casos, la heterogeneidad en los exponentes de singularidad z es capaz de inducir comportamientos colectivos periódicos no presentes en la dinámica local de los mapas.



- La variabilidad temporal de los parámetros locales distribuidos, induce comportamientos colectivos ordenados en una red dinámica.