Crecimiento y formación de fases en medios heterogéneos.

(Defensa.)

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Introducción.

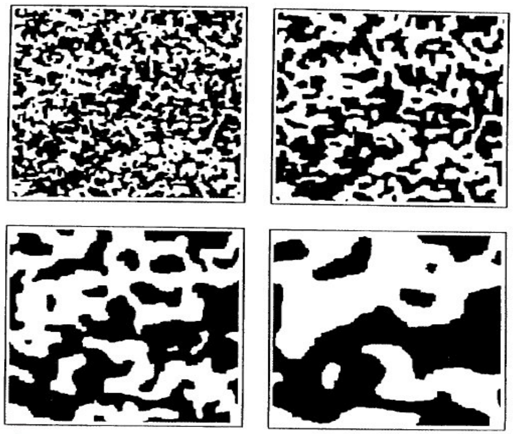
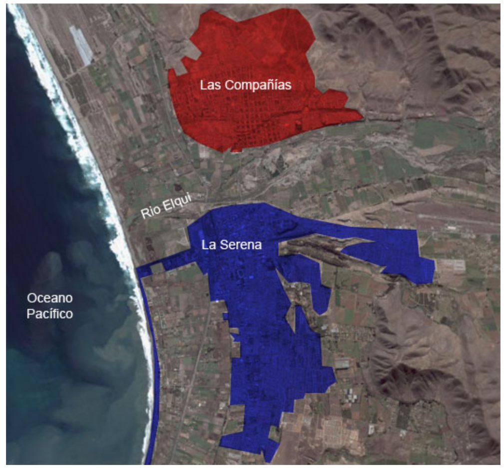
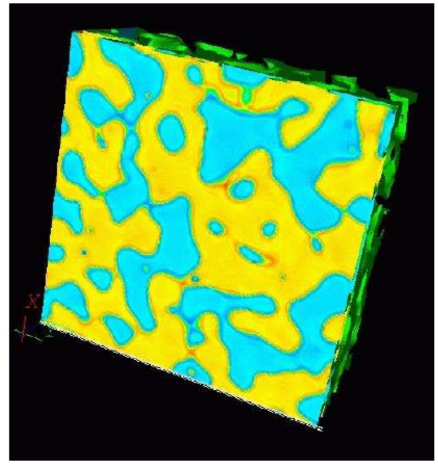


Figure 2 Monte Carlo simulation of domain growth in the $d = 2$ Ising model at $T = 0$ (taken from Kissner [8]). The system size is 256×256, and the snapshots correspond to 5, 15, 60 and 200 Monte Carlo steps per spin after a quench from $T = \infty$





1. Crecimiento y fomación de fases en RMA sobre medios heterogéneos.
2. Descripción de sistemas sociales multiculturales en medios heterogéneos.
3. Difusión de Fluidos, estabilización de interfases y formación de fases de fluidos binarios en medios congestionados.

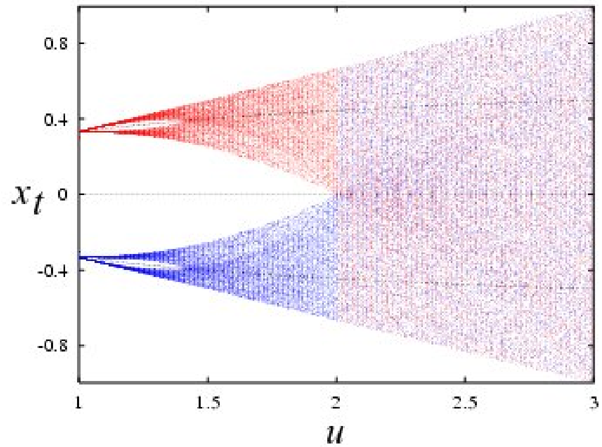
The Model of CML.

Consideramos un sistema de mapas acoplados definidos:

$$x_i(t+1) = (1-\epsilon) f(x_i(t)) + \frac{\epsilon}{n_i} \sum_{j \in \nu_i} f(x_j(t)), \tag{1}$$

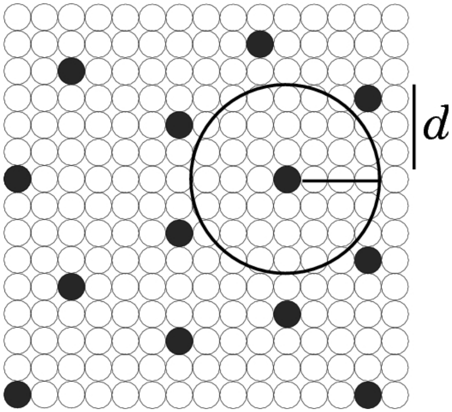
Donde ν_i es el conjunto de vecinos de cada sitio i, n_i es la cardinalidad de ν_i, ϵ es el parámetro de acoplamiento y $f(x(t))$ es la función no lineal que representa la dinámica local.

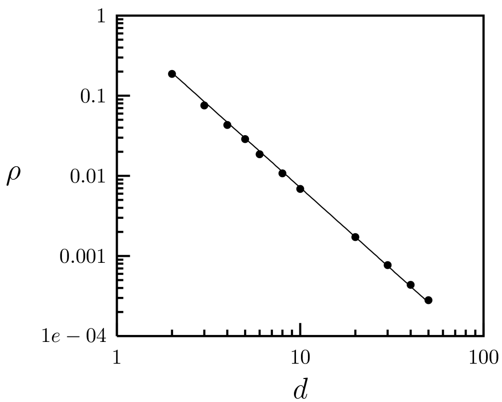
$$f(x) = \begin{cases} -2\mu/3 - \mu x & if \quad x \in [-1, -1/3] \\ \mu x & if \quad x \in [-1/3, 1/3] \\ 2\mu/3 - \mu x & if \quad x \in [1/3, 1]. \end{cases} \tag{2}$$

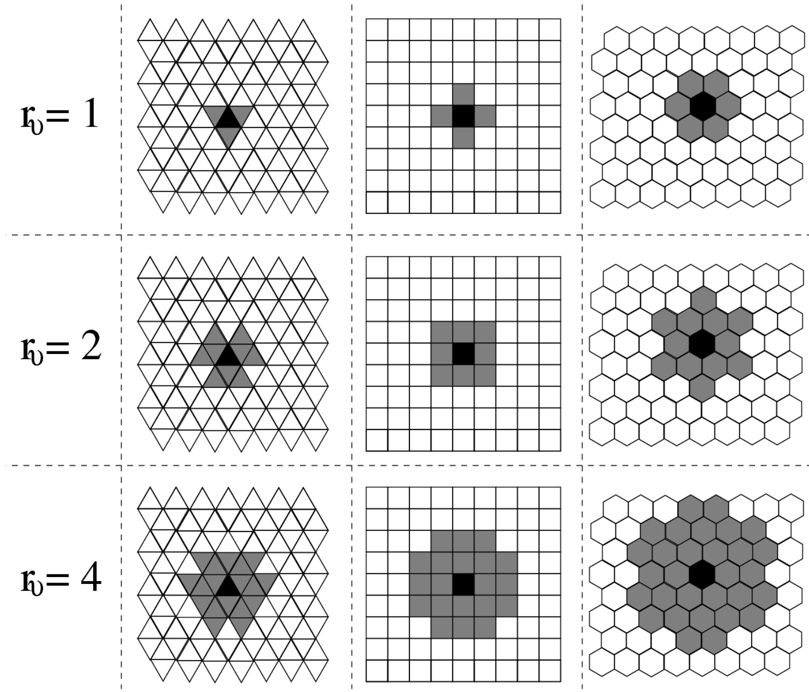


La fase es definida por $\sigma_i(t) = \mathrm{sign}(x_i(t))$ y $\mu = 1{,}9$.

El sustrato.





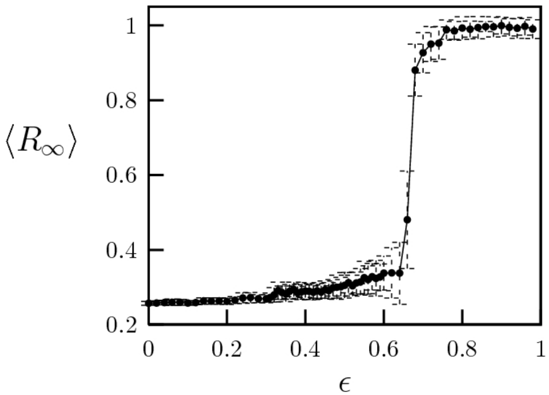


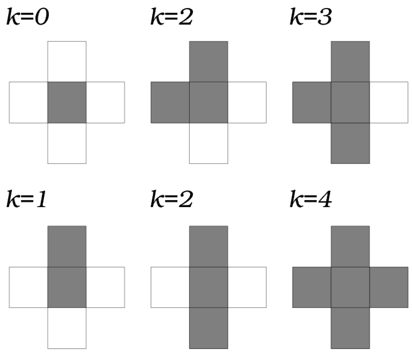
Resultados.

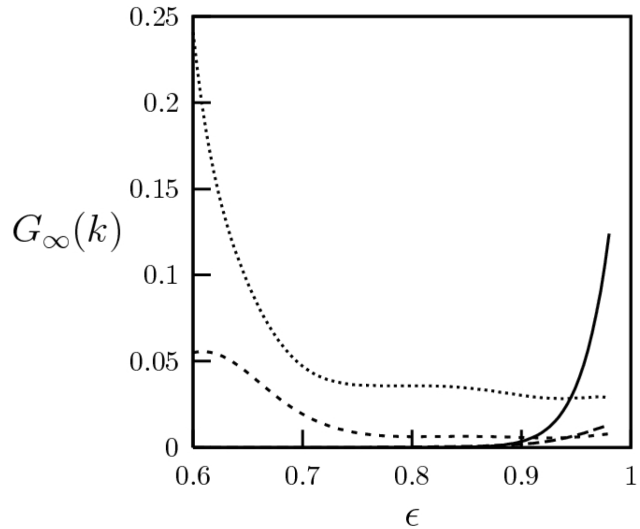
$$R_t = \frac{1}{N} \sum_{r=1}^{L/2} \sum_{i,j} \delta_{r_{ij},r} \delta_{\sigma_t^i, \sigma_t^j} , \qquad (3)$$

$$G_t(k) = \frac{1}{N} \sum_{i=1}^{N} F_t^i(k) \qquad (4)$$

$$F_t^i(k) = \begin{cases} 1, \ \text{si} \ \sum_{j \in N_i^v} \delta_{\sigma_t^i, \sigma_t^j} = k, \\ \\ 0, \ \text{si} \ \sum_{j \in N_i^v} \delta_{\sigma_t^i, \sigma_t^j} \neq k \end{cases} ; \qquad (5)$$

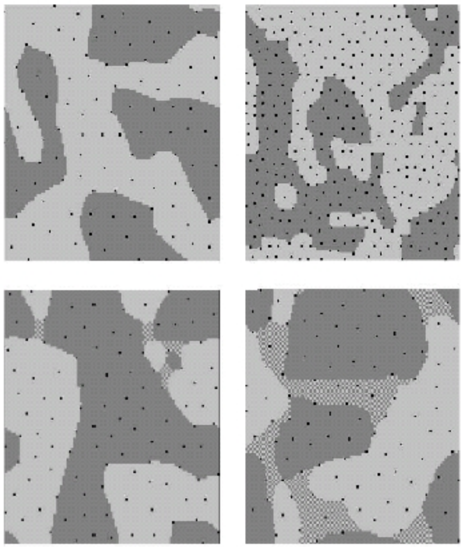


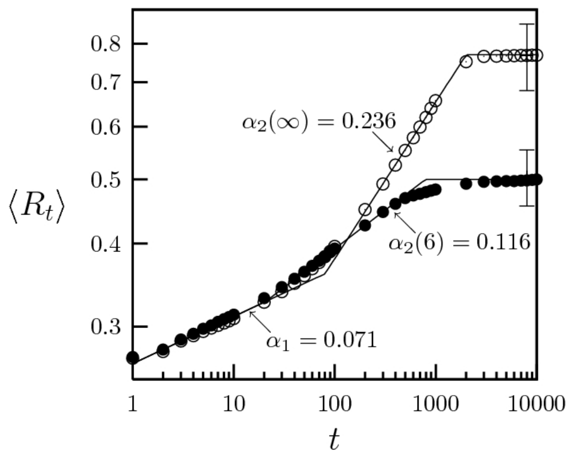


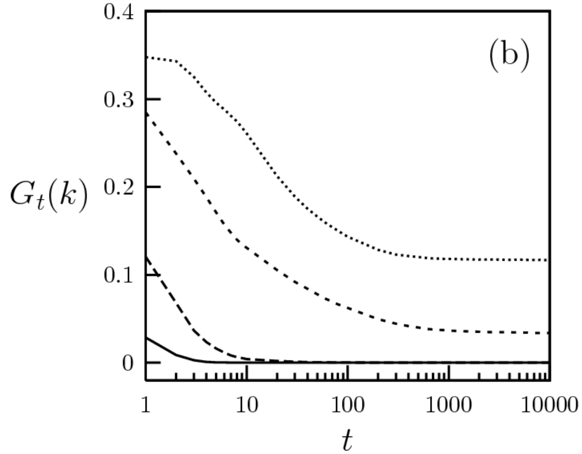


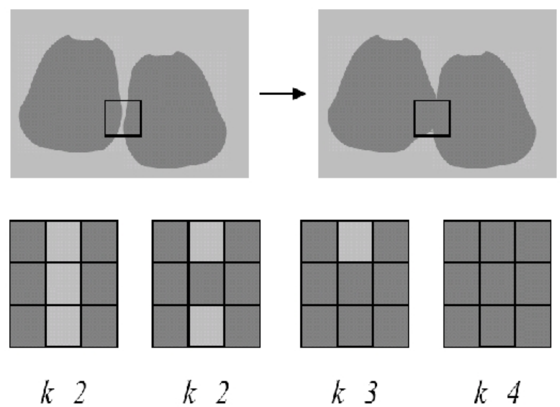
Con impurezas.

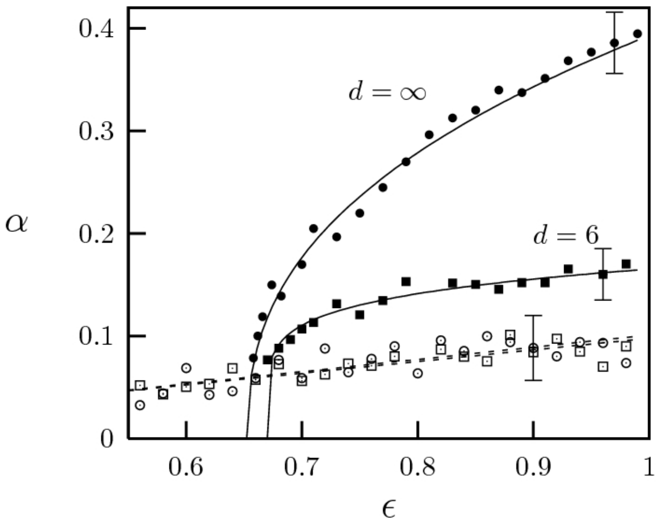
$$\langle R_t \rangle \sim t^{\alpha} \qquad (6)$$





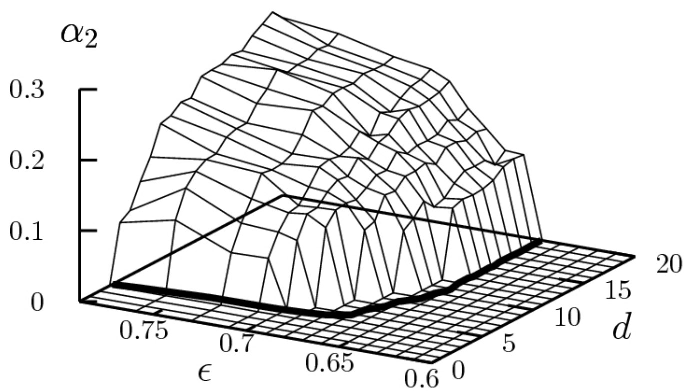


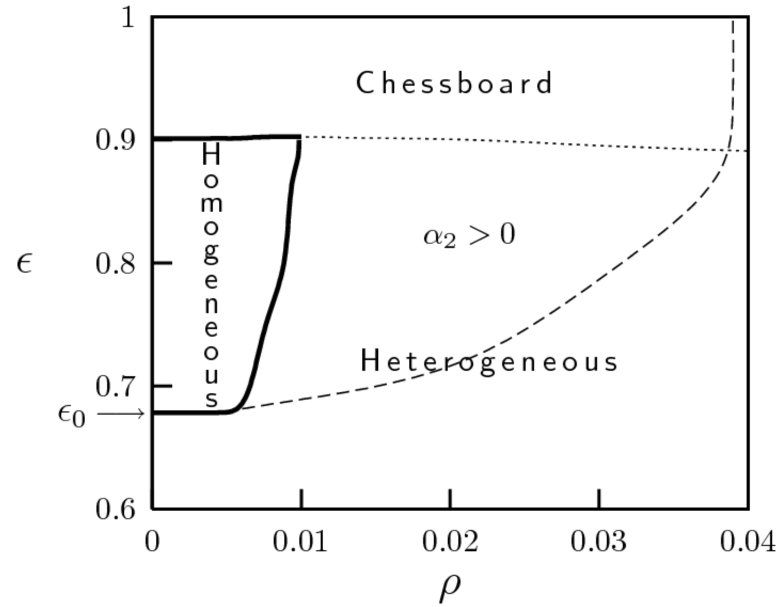




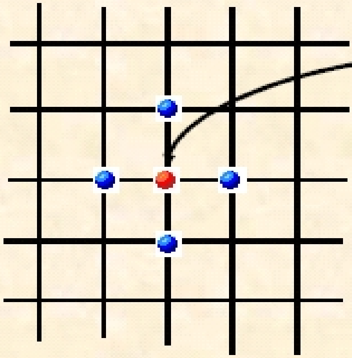
$$\alpha_2 \sim (\epsilon - \epsilon_c)^{\gamma} \tag{7}$$

$$\langle R_\infty \rangle \sim (d - d_c)^{\beta(\epsilon)} \tag{8}$$





Axelrod's agents based model: interaction



agent i

agent i's neighbors

$$\begin{pmatrix} \sigma_{i1} \\ \sigma_{i2} \\ \vdots \\ \sigma_{iF} \end{pmatrix}$$

F = # **Features**

q = # **Traits** per feature

$\sigma_{if} \in \{0, \ldots, q\text{-}1\}$

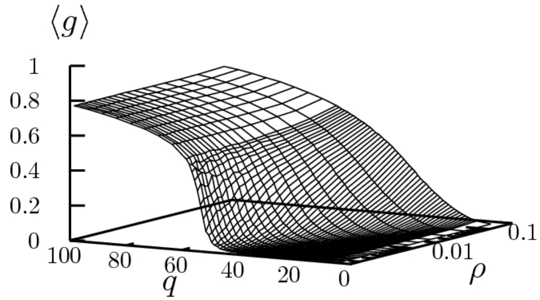
F=3; q=10 $\qquad$ q^F (10^3) *equivalent* cultural options.

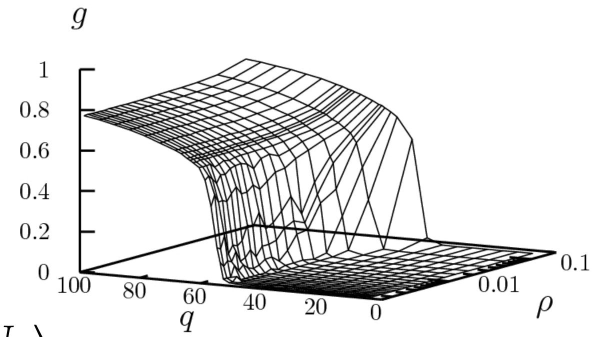
$$\begin{pmatrix} 0 \\ 0 \\ \boxed{7} \end{pmatrix} \begin{pmatrix} 5 \\ 9 \\ \boxed{7} \end{pmatrix} \longrightarrow \begin{pmatrix} \boxed{5} \\ 0 \\ \boxed{7} \end{pmatrix} \begin{pmatrix} \boxed{5} \\ 9 \\ \boxed{7} \end{pmatrix}$$

Mechanism of local convergence:

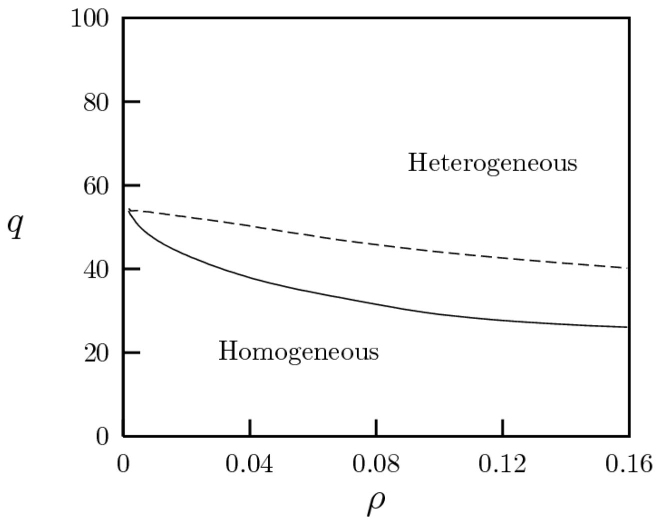
Prob to interact =

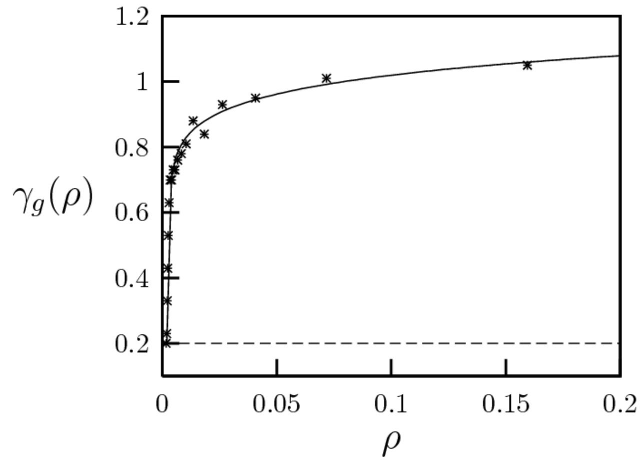
$$\frac{\text{Common features}}{F} = \frac{1}{3}$$



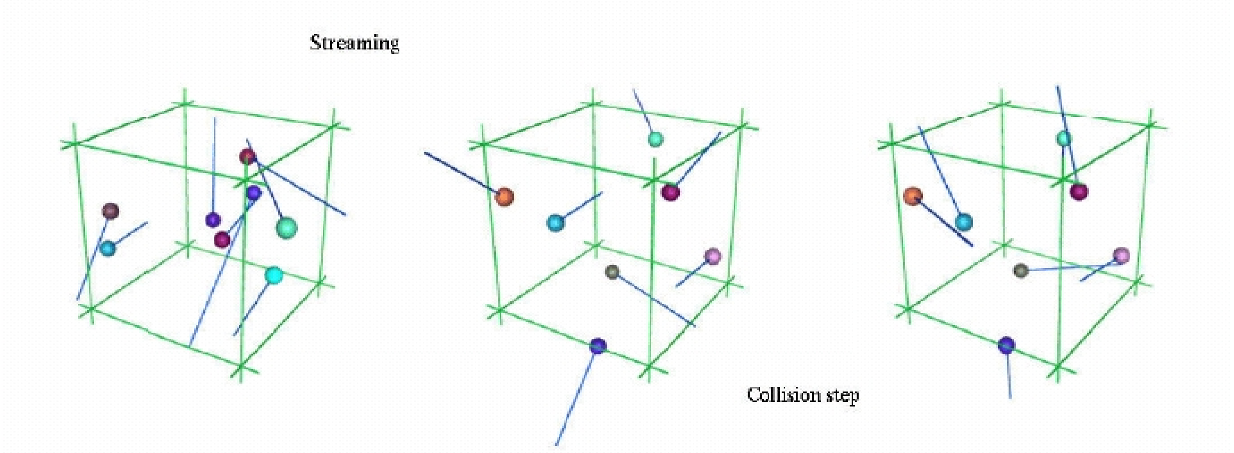


$$g = \frac{\langle N_g \rangle}{N} \tag{9}$$





Dinámica de colisión de multipartículas.



Consideremos N partículas con masa m.

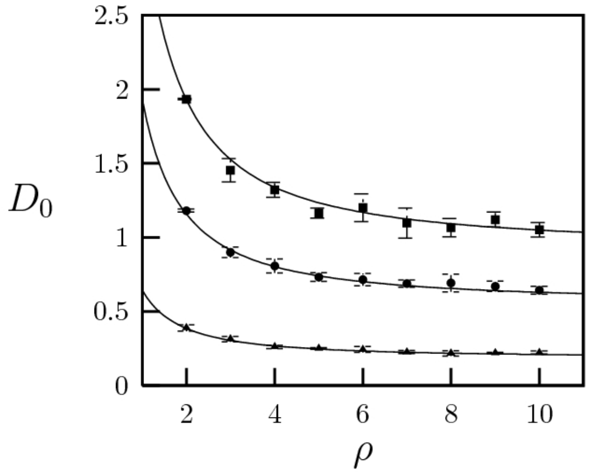
$$\mathbf{x}^s(t+\tau) = \mathbf{x}^s(t) + \mathbf{v}^s(t)\tau, \quad (10)$$

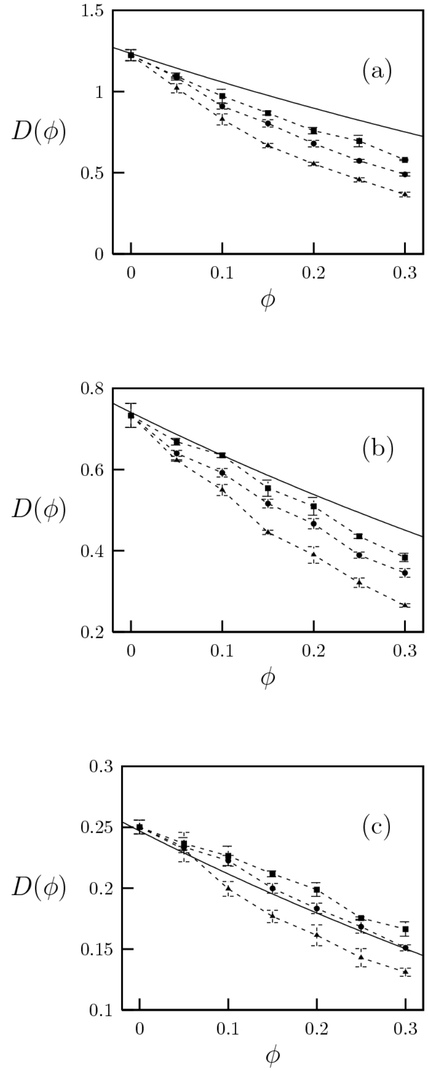
$$V_\xi = \frac{1}{n_\xi}\sum_{i|\mathbf{x}\in\nu} \mathbf{v}'_i, \quad (11)$$

$$\mathbf{v}^s = V_\xi + \widehat{\omega}_\xi(\mathbf{v}^{s\prime} - V_\xi). \quad (12)$$

$$D_0 = \frac{1}{2}\langle v_x v_x\rangle + \sum_{l=1}^{\infty}\langle v_x v_x(l_\tau)\rangle, \quad (13)$$

$$D_0 \approx \frac{k_b T}{2m}\left(\frac{2\rho + 1 - e^{-\rho}}{\rho - 1 + e^{-\rho}}\right) \quad (14)$$





$$\phi \equiv 4\pi N_{obs}\sigma^3/3\mathcal{V} \qquad (15)$$

$$\frac{\partial}{\partial t}\rho(\mathbf{r},t) = D_0\nabla^2\rho(\mathbf{r},t) - \nabla\cdot\mathbf{J}(\mathbf{r},t), \quad (16)$$

$$D(\phi) = D_0 + \triangle D_0 = 2\,D_0\frac{1-\phi}{2+\phi}, \quad (17)$$

Dinámica de colisión multipartículas para dos especies

$$\Theta_s^{\varsigma}\Theta_s^{\varsigma'} = \delta_{\varsigma}\delta_{\varsigma'};\;\; \textstyle\sum_{\varsigma}\Theta_s^{\varsigma} = 1;\;\; N_{\varsigma} = \displaystyle\sum_{s=1}^{N}\Theta_i^{\varsigma}. \tag{18}$$

$$V_{\xi}^{\varsigma}(t) = \frac{1}{n_{\xi}^{(\varsigma)}(t)}\sum_{s|\mathbf{x}\in\Omega}\theta_s^{\varsigma}\mathbf{v}_s(t); \tag{19}$$

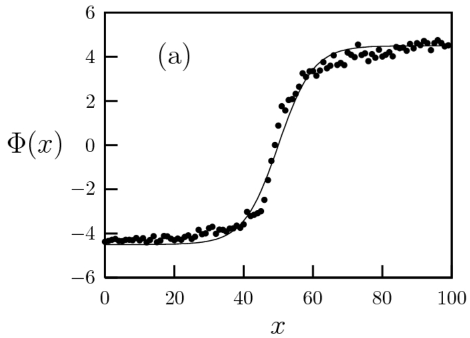
$$V_{\xi}(t) = \frac{\sum_{\varsigma} n_{\xi}^{(\varsigma)} m_{\varsigma} V_{\xi}^{(\varsigma)}(t)}{\sum_{\varsigma} n_{\xi}^{(\varsigma)} m_{\varsigma}}. \tag{20}$$

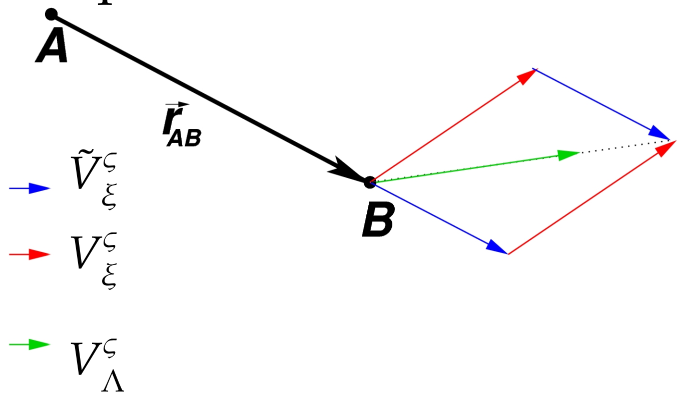
$$\mathbf{v}^{s\prime\prime} = V_{\xi} + \widehat{\omega}_{\xi}(\mathbf{v}^{s} - V_{\xi}) \tag{21}$$

$$\mathbf{v}^{S*} = \sum_{\varsigma}\Theta_s^{\varsigma}\left(V_{\xi}^{\prime\prime(\varsigma)} + \widehat{\omega}_{\xi}^{\varsigma}\left(\mathbf{v}^{s\prime\prime} - V_{\xi}^{\prime\prime(\varsigma)}\right)\right), \tag{22}$$

$$\mathbf{v}^{S*} = V_{\xi} + \widehat{\omega}_{\xi}(\Theta_{\xi}^{\varsigma}V_{\xi}^{(\varsigma)} - V_{\xi}) + \sum_{\varsigma}\Theta_s^{\varsigma}\left(\widehat{\omega}_{\xi}^{\varsigma}\widehat{\omega}_{\xi}\left(\mathbf{v}^{s} - V_{\xi}^{\prime\prime(\varsigma)}\right)\right), \tag{23}$$

Inclusión de la repulsión para dos especies.





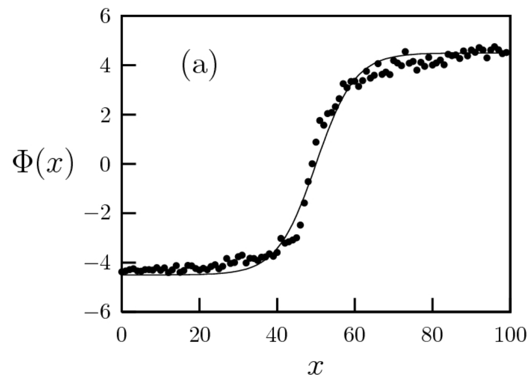
$$\Phi(x) = \left\langle n^{\varsigma}(x) - n^{\varsigma'}(x) \right\rangle_{\xi_x} \quad (28)$$

$$r^{c}_{\varsigma\varsigma'} = \begin{cases} 2{,}5\, r_{\varsigma} & si \quad \varsigma = \varsigma' \\ 2^{1/6}\, r_{\varsigma} & si \quad \varsigma' \neq \varsigma'. \end{cases} \quad (24)$$

$$V^{\varsigma}_{\Lambda} = \kappa n^{(\varsigma')}_{\xi} m_{\varsigma'} \widehat{r}_{AB} + V^{\varsigma}_{\xi} \quad (25)$$

$$\tilde{V}^{\varsigma}_{\xi} = \frac{V^{\varsigma}_{\Lambda}}{|V^{\varsigma}_{\Lambda}|} |V^{\varsigma}_{\xi}| \quad (26)$$

$$\mathbf{v}^{s^*} = \sum_{\varsigma} \Theta^{\varsigma}_{s} \left(\tilde{V}^{\varsigma}_{\xi} + \widehat{\omega}^{\varsigma}_{\xi} \left(\mathbf{v}^{s} - V^{\varsigma}_{\xi} \right) \right) , \quad (27)$$



$$\Phi(x) = \Phi_{eq} \tanh \frac{x}{\zeta} , \quad (29)$$

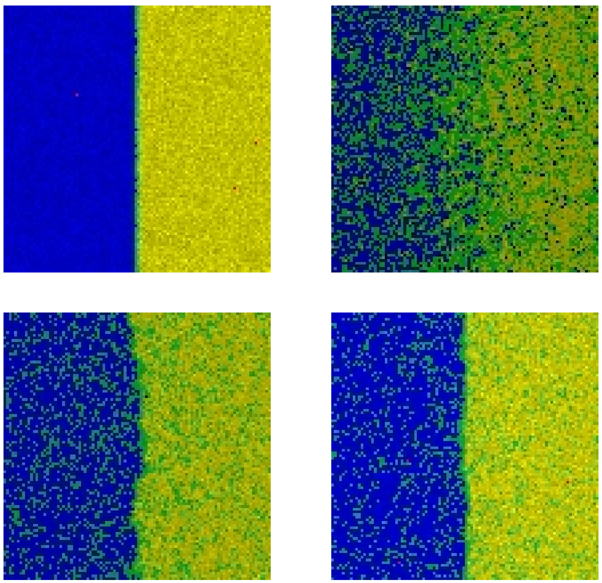
Para el modelos de Ising:

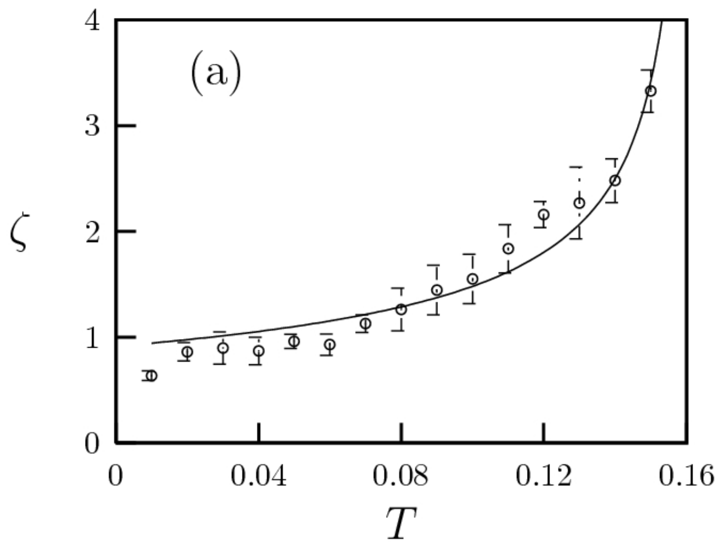
$$F_{GL}[\Phi(\mathbf{r})] = \int d\mathbf{r}\Big[-\frac{a_2}{2}\Phi(\mathbf{r})^2 + \frac{a_4}{4}\Phi(\mathbf{r})^4 + \frac{K}{2}|\nabla\Phi(\mathbf{r})|^2\Big] \qquad (30)$$

$$a_2 = \frac{S-4T}{2\,a^3}, \qquad a_4 = \frac{4\,T}{3\,a^3}, \qquad K = \frac{S}{4\,a}, \qquad (32)$$

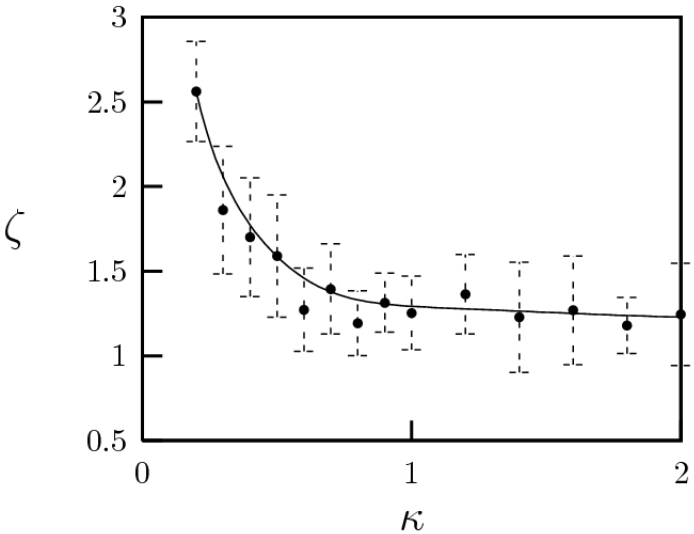
$$\Phi(x) = \sqrt{\frac{a_2}{a_4}}\tanh\frac{x}{\zeta}, \qquad (31)$$

$$\zeta = \sqrt{\frac{2\,K}{a_2}} \sim \frac{1}{(T_c - T)^{1/2}} \qquad (33)$$





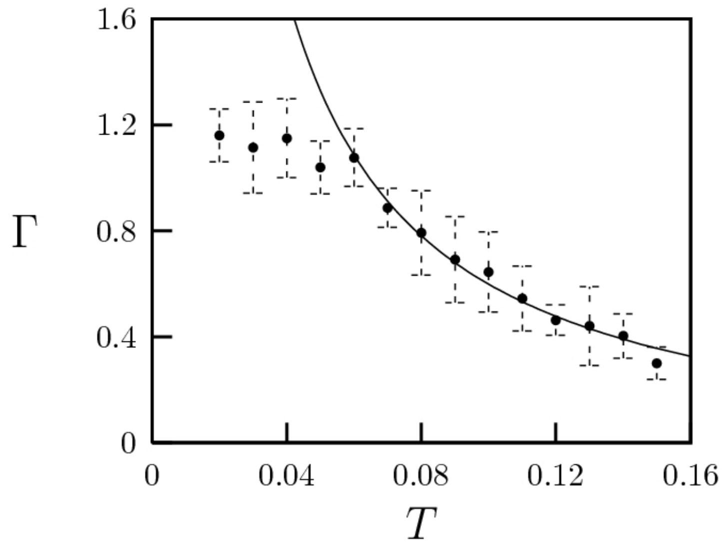
$$\Gamma = K \int dz \left(\frac{\partial \Phi(x)}{\partial x} \right)^2 . \qquad (36)$$



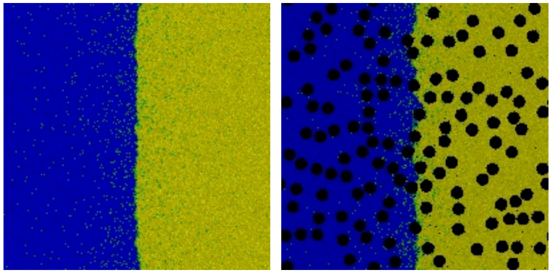
$$\Gamma = \frac{4\, K\, \Phi_{eq}^2}{3\, \zeta} , \qquad (37)$$

$$\Gamma = \frac{K\,(T_c - T)}{2\, T\, a\, \zeta} \sim \frac{(T_c - T)^{3/2}}{T} . \qquad (38)$$

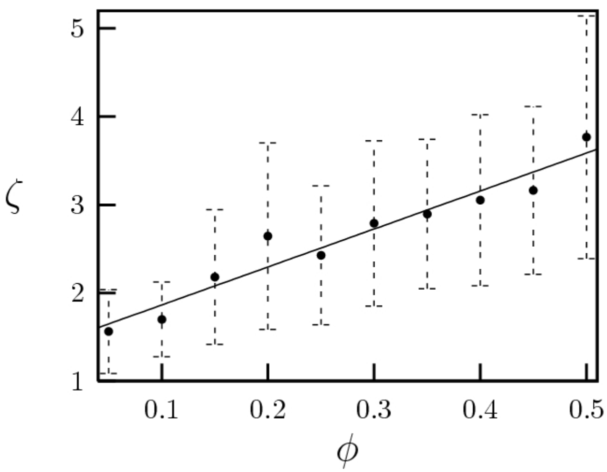
$$\begin{aligned} \Gamma &= \int dx \Big(f_o(\Phi(x)) - f_o(\Phi_{eq}) \\ &+ \frac{K}{2} \left(\frac{\partial \Phi(x)}{\partial x} \right)^2 \Big) , \end{aligned} \qquad (34)$$

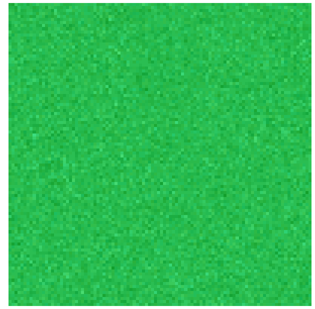


$$f_o(\Phi(x)) - f_o(\Phi_{eq}) = \frac{K}{2} \left(\frac{\partial \Phi(x)}{\partial x} \right)^2 , \qquad (35)$$



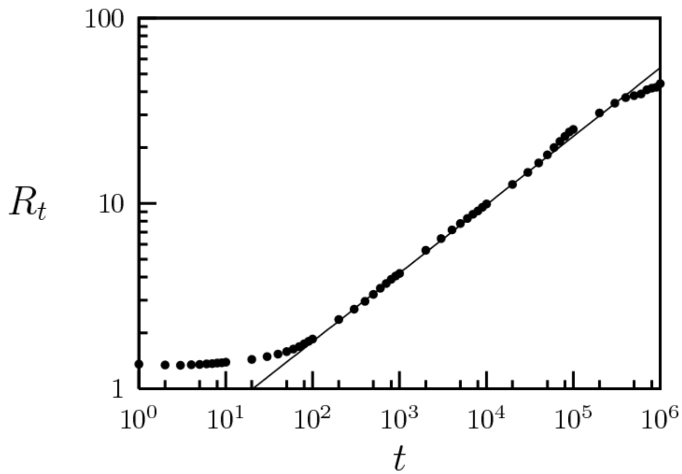
$$\Phi(x) = \sqrt{\frac{a_2}{a_4}} \tanh \frac{x}{(\zeta\,(1-\phi))}\,, \tag{39}$$

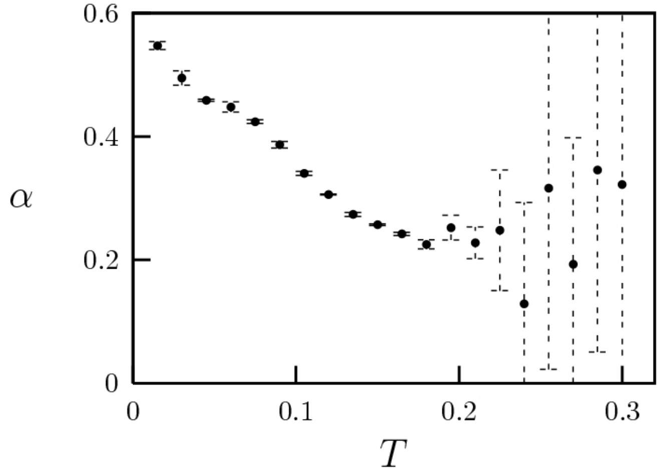




$$\langle \Phi(\mathbf{r}_{\xi'}, t)\Phi(\mathbf{r}_{\xi}, t)\rangle_{\xi,\xi'} = C\,[R_t, t] = 0 \tag{40}$$

$$R_t \sim t^{\alpha} \tag{41}$$




0.6
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0
α
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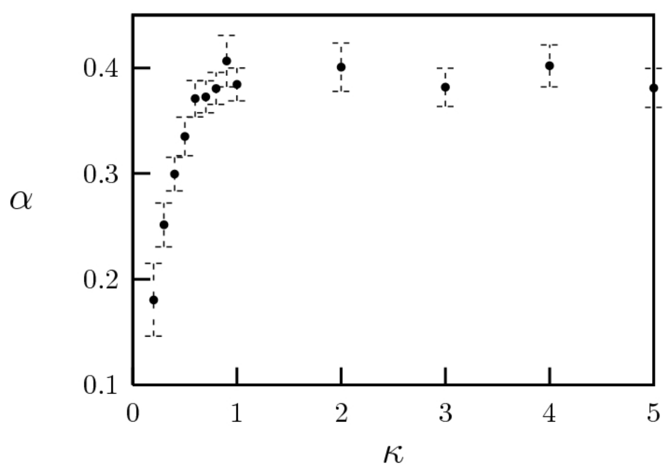
0.4
0.3
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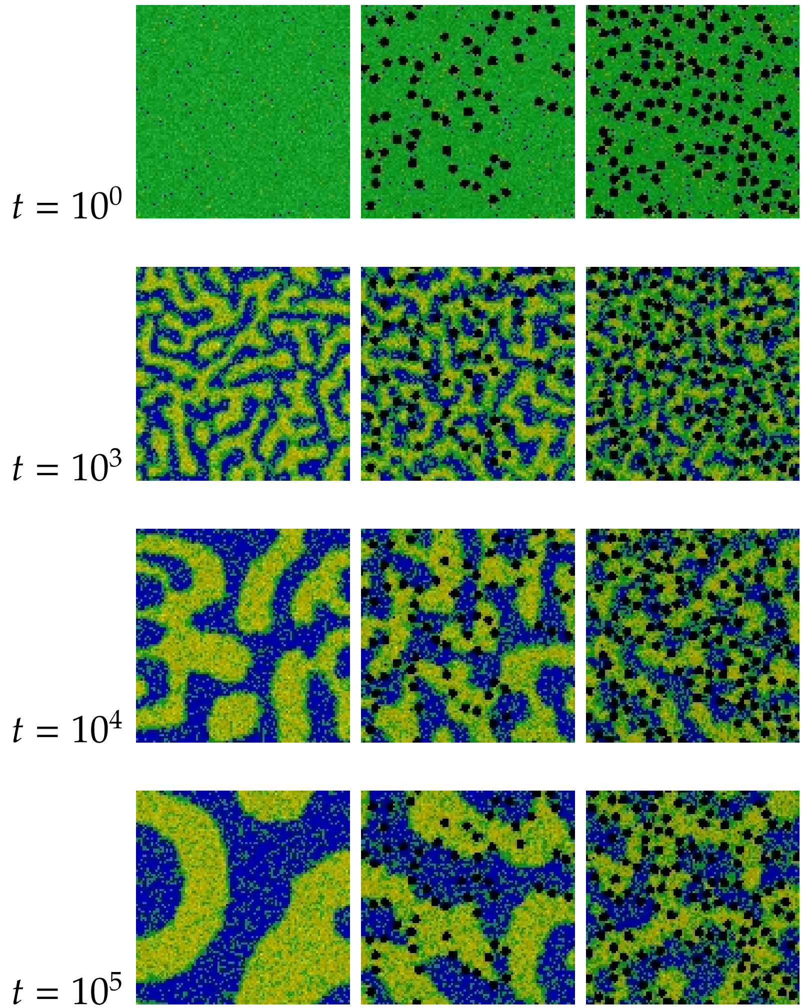
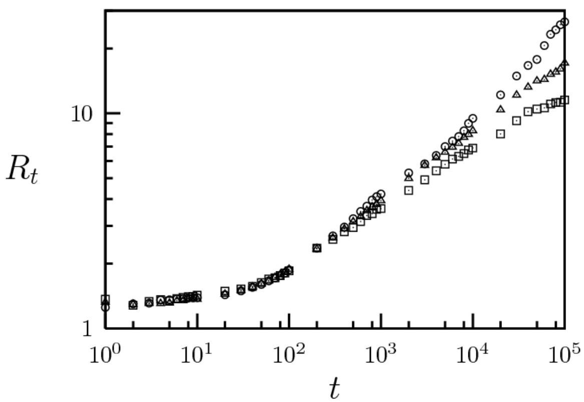
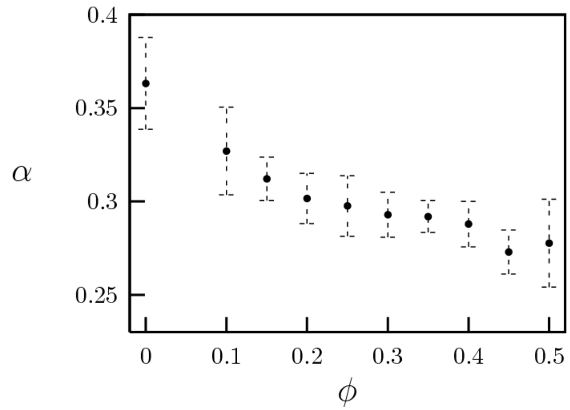

$t = 10^0$
$t = 10^3$
$t = 10^4$
$t = 10^5$





Conclusiones:

- Se mostró que la velocidad con la cual los dominios de fase crecen, es más baja cuando las impurezas están presentes en el sistema.
- El tamaño promedio de los dominios resultantes en el estado inhomogéneo de el sistema decrece cuando la densidad de impurezas aumenta.
- La topología del medio puede jugar un rol decisivo en el determinación del comportamiento colectivo emergente.
- Las inhomogeneidades espaciales también pueden ser empleados como un mecanismo de selección para modular el tamaño de los patrones en

sistemas espaciotemporales.

- Las impurezas tienen un efecto sobre las propiedades críticas, de pasar de un estado heterogéneo a uno homogéneo.
- Las tres técnicas de modelado mostraron ser eficientes para el estudio del crecimiento y formación de fases en medios heterogéneos.