Article

Journal of Conflict Resolution
00(0) 1-22

DOI: 10.1177/0022002713498708
jcr.sagepub.com

Tolerating the Intolerant: Homophily, Intolerance, and Segregation in Social Balanced Networks

Fernando Aguiar[1] and Antonio Parravano[1,2]

Abstract

We model a community of individuals whose relationships are governed by the rules of the so-called *Heider balance theory*, but modified to address the impact of tolerating intolerant individuals. To consider tolerance toward a different group, the elements are assigned one of the two flags, *A* or *B*, and the elements of each group can be tolerant or intolerant. Two additional parameters, *p* and *q*, respectively, characterize the propensity of elements to cooperate and the propensity of tolerants to reject intolerant attitudes. We find that (1) parameter *q* does not affect the degree of conflict at the micro level, but has an important influence on the degree of conflict in the whole system; (2) segregation into two cliques occurs whenever there exists intolerants in both groups; (3) when intolerants are present in only one of the groups, segregation can be avoided for appropriate combinations of parameters *p* and *q* that depend on the fraction of intolerants and the size of the groups; (4) as the size of the system increases, two balanced solutions dominate: segregation into two cliques or the isolation of intolerants; and (5) endemic partially balanced configurations are observed in large systems.

[1] Instituto de Estudios Sociales Avanzados (IESA-CSIC), Córdoba, Spain
[2] Centro de Física Fundamental, Universidad de Los Andes, Mérida, Venezuela

Corresponding Author:
Antonio Parravano, Instituto de Estudios Sociales Avanzados, Campo Santo de Los Martires, 7, 14004 Cordoba, Spain.
Email: parravan@ula.ve

Keywords
social networks, social dynamics, segregation, homophily, intolerance

Homophily and Intolerance

Intolerance has always been a major source of conflict and social segregation. Rejecting or excluding others based on their appearance, culture, beliefs, and other characteristics is such a widespread social behavior that one can say it has been present and continues to be so in all societies. For this reason, intolerance has been approached from biological, sociological, philosophical, and historical perspectives in an attempt to understand its deep roots and extension in human behavior (Noël 1994). To the best of our knowledge, however, agent-based simulation models have not paid enough attention to the phenomenon of the tolerance of intolerance in spite of its social importance and conflict-allowing nature. Given the obvious interest and explanatory power of the Schelling and Axelrod models, special attention has been paid mainly to the mechanisms by which homophily gives rise to segregation and cultural diversity (Schelling 1971; Axelrod 1997; Castellano, Marsilli, and Vespignani 2000; McPherson, Smith-Lovin, and Cook 2001; Centola et al. 2007; Parravano, Rivera-Ramírez, and Cosenza 2007; Gracia-Lázaro et al. 2009; Abdou and Gilbert 2009). Nevertheless, in understanding homophily as people's preference for interacting with those with similar traits, it seems clear that there is a close relationship between homophily and intolerance. This relation is manifested when a tolerant person is faced with the dilemma of choosing between establishing a positive relationship with a tolerant individual of a dissimilar group or establishing a positive relationship with an intolerant group member. In the first case, the intolerant disapproves the established link, leading necessarily to a negative relationship with his or her equal. In the second case, the negative relationship toward the other-group individual is endorsed by the intolerant in-group member and promotes a positive relationship between them. Hereafter, we refer to this situation as "the dilemma of the tolerant," or the T-dilemma for short.

The so-called *contact hypothesis*—the idea that contact between members of different races could favor positive attitudes and reduce racial hostility (Sigelman and Welch 1993; Dixon, Durrheim, and Tredoux 2005)—has proved to be insufficient in explaining all the existing evidence, thus highlighting the complex trade-off between homophily and tolerance that all societies have to face. While multicultural blacks and whites, or Christians and Muslims, for example, are able to transcend homophily to maintain links with the dissimilar, in many cases racial and cultural segregation can be maintained by the homophilic pressure of intolerants (racists or xenophobes). What then is the social impact of the behavior adopted at the individual level by tolerant individuals facing the T-dilemma? This is the main issue addressed in the present study.

To analyze the effect of the interplay between homophily and tolerance, we consider a network of fully connected[1] individuals that belong to one of the two

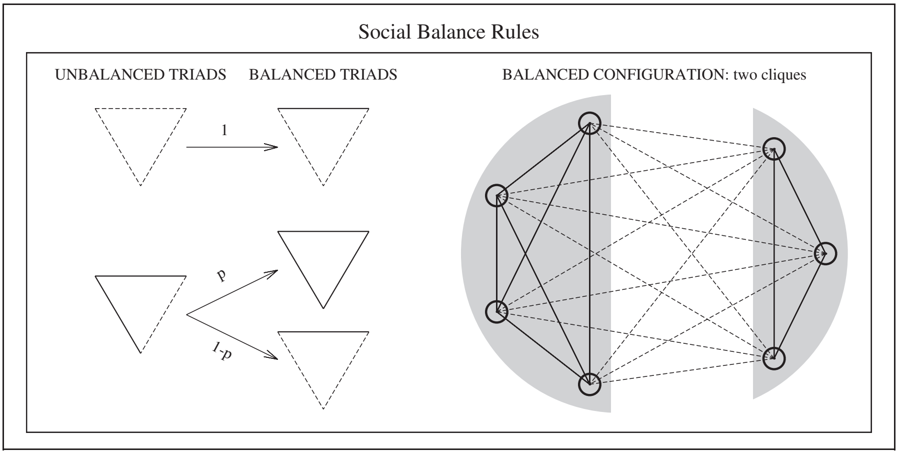

Figure 1. Schematic evolution of Heider's triads.

Note: Solid edges represent a friendly relationship and dashed edges represent a rival relationship. The triads at the left are the two possible unbalanced (unstable) configurations and evolve to the two possible balanced configurations at their right. The configuration of the seven-element system at the right is balanced since all possible triads are balanced. The individuals are segregated into two antagonistic cliques: within the same clique there are only friendly links, while the individuals from different cliques are rivals.

groups (A or B) and have one of the two social attitudes (tolerant or intolerant). Additionally, in order to model the behavior of individuals who are subjected to the T-dilemma, we assume that the interaction among the individuals occurs in triads rather than in pairs. To do so, we use a modified version of Heider's balance theory (Heider 1946, 1958; see also Cartwright and Harary 1956; Hummon and Doreian 2003; Antal, Krapivsky, and Redner 2005; Gawroński, Gronek, and Kułakowski 2005; Kułakowski, Gawroński, and Gronek 2005; Marvel, Strogatz, and Kleinberg 2009) because this theory takes into account a central aspect in the process of social segregation: the interference of third-party actors in pair relations.

Before considering the T-dilemma situation, let us describe the rules that govern the evolution of the links in homogeneous triads. Specifically, they are triads of three tolerant individuals regardless of their group membership, or triads of three individuals of the same group regardless of whether they are tolerants or intolerants. For these homogeneous triads, we adopt the simplified Heider rules proposed by Cartwright and Harary (1956) shown in schematic form in Figure 1, where a solid line represents a friendly link and a dashed line a rival relation. The top-left situation shown in Figure 1 corresponds to the evolution toward alliance in the face of a common enemy. The bottom-left situation represents the evolution based on the principle that if you have two friends who are rivals, sooner or later one of the two things will happen: either the rivals will reconcile (all-friends solution) or you will end your friendship with one of them. Note that this model omits any description of the internal state of the individuals, that is, all the information is contained in the links. If we define $S_{i,j}$ as the state of the relation between elements i and j, and $S_{i,j} = +1$ if i and j are friends and $S_{i,j} = -1$ if i and j are rivals, then a triad (i, j, k) is unbalanced if the product $S_{i,j}S_{i,k}S_{k,j} = -1$, or equivalently if the number of rival links is odd. The triads at the left of Figure 1 are the two possible unbalanced (unstable) configurations and evolve to the two possible balanced configurations at their right. Label p refers to the probability of evolving to the three-friend balanced configuration, and is referred to here as the *consensus parameter*.

The simplest collective stable state is the homogeneous solution with all $S_{i,j} = +1$ (i.e., the all-friends solution where all triads are balanced in the three-friends configuration). The other stable situation is a two rival clique with $S_{i,j} = +1$ for all members i and j within the same clique and $S_{i,j} = -1$ when the members i and j belong to different cliques as shown on the right-hand side of Figure 1. In those situations, the network is completely balanced since each triad is balanced. Segregation in two groups is also observed in models in which the time and the link strength are continuous variables (Kułakowski, Gawroński, and Gronek 2005; Marvel et al. 2011).

A real example of the segregation induced by the mechanisms involved in the triad balance theory was given by Zachary (1977) in his study of the two-year splitting process occurring in a group of thirty-four members of a karate club due to the conflict between the instructor and the administrator of the club.

To include the dynamics of triads that are subjected to the T-dilemma, we have introduced an additional parameter that represents the propensity of tolerant individuals to refuse the discriminating attitudes of their peers. The distinction of four types of individuals depending on their flag (A or B) and their attitude (tolerant or intolerant), together with the inclusion of triad dynamics in both types of triads (homogeneous triads and triads subjected to the T-dilemma) allows to analyze the necessary conditions for segregation to occur in the context of this model, that is, either the isolation of intolerants (II) or segregation into groups A and B.

The rules of evolution of the links are described in the next section, while the model behavior in the space of parameters is described in the third section. In the fourth section, we discuss the limitations of the model to be implemented in large networks and present the results for a modified model that can be used to simulate larger systems in Appendix. In the fifth section, we describe various related works and discuss some possible refinements of our model. Finally, the overall results are discussed in the sixth section.

The Model

Let us consider a system of N elements that can be of four types, AT, AI, BT, BI, depending on their flags, A and B, and their attitudes, T (for tolerant) and I (for intolerant). The flag and the attitude of elements remain unchanged during a simulation and their values are established at the beginning of each simulation. Each pair of elements (i, j) is characterized at a discrete time t by a link state $S^t_{i,j}$ that can take one of the two values: $+1$ or -1, which are associated to a friend link or to a rival link, respectively. To keep the model as simple as possible, we do not include learning considerations regarding individuals change in attitude or change in appearance to mislead the individuals with a different flag. For the same reason, we consider symmetric links ($S^t_{i,j} = S^t_{j,i}$) and that the network is fully connected. Possible refinements of this model are discussed in the fifth section.

In most agent-based models, the rules used to update the state of agents depend on the interaction between pairs of elements and/or on the mean state of their neighbors. In Heider's balance theory, however, the evolution of the links is determined by rules that apply to triads of elements. This is a fundamental and convenient property of this model, since in many situations the relation between two individuals depends on their relation with a third actor, and this micro-level triad relation could be essential to reproduce some macro-level social facts.

Since there are four types of elements, there are twenty possible triads[2] that can be reduced to ten when, as in our case, the rules that govern the evolution of the links remain the same when flags A and B are interchanged. To establish the rules that govern the evolution of the various types of triads, we assume that tolerant individuals do not take into account their partner's flag to establish their link and that an intolerant individual never establishes a friend link with an individual having a different flag.

These rules are summarized in Figure 2. The triads can be grouped into three categories:

1. The first ten triads in Figure 2 are formed by three tolerant individuals and/or by three individuals with the same flag. Since intolerance is irrelevant for updating the triads in this category, we assume that these triads follow the balance rules described in Figure 1. The parameter p in Figure 2 plays the same role as in Figure 1, and has the same meaning: propensity to establish a positive link. We therefore continue to refer to it as the *consensus parameter*.
2. Triads 11 and 12 are formed by two elements with the same flag; one of which is tolerant and the other intolerant. The element possessing the other flag is tolerant. In these triads, the T-dilemma is present since a tolerant individual has to choose between establishing a positive relation with a tolerant individual of a dissimilar group or establishing a positive relation with an in-group, intolerant member. That is, there are only two possible stable configurations, one in which the two tolerant elements are friends but rivals with the intolerant one and another in which the two elements with the same flag are friends but rivals with the element with the other flag. The first configuration occurs with a probability q. We term this parameter *intolerance to intolerants*. Note that parameter q measures the prevalence of intolerance to intolerants over homophily. For $q = 0$, homophily always dominates (the positive link is always established between a tolerant–intolerant pair with the same flag), and for $q = 1$ intolerance to intolerants always dominates (the tolerant always establishes the friendly link with the other tolerant

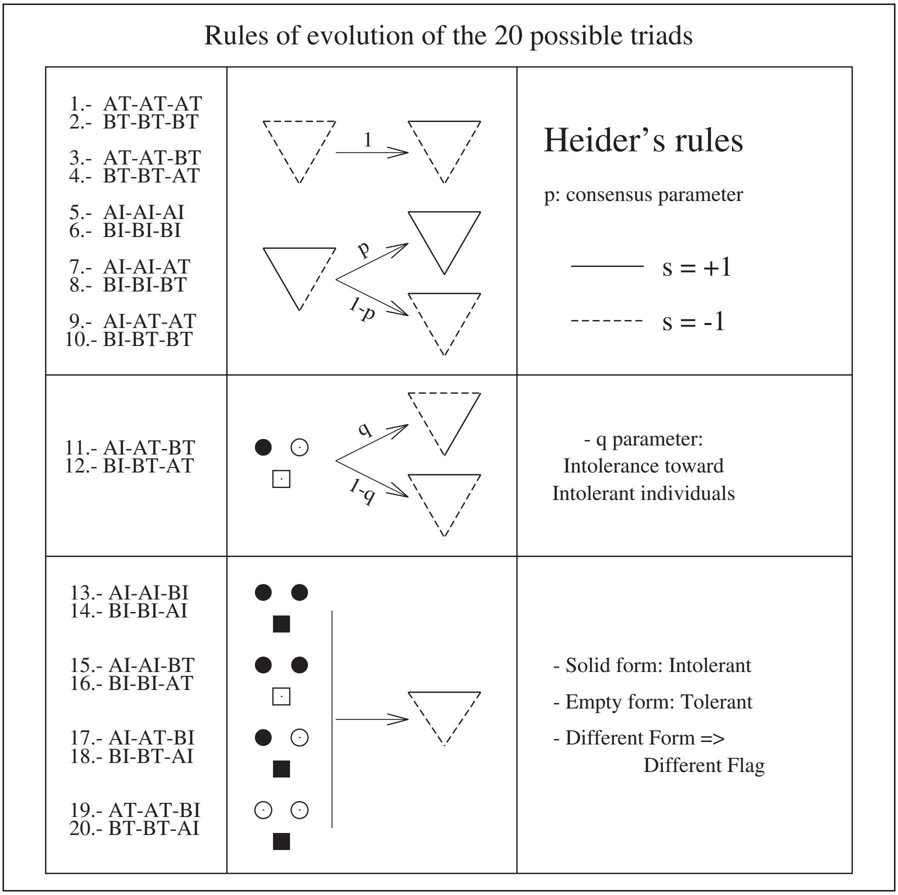

Figure 2. Schematic evolution of the twenty possible triads.
Note: Solid edges represent friend links ($S_{i,j} = +1$) and dashed edges represent a rival link ($S_{i,j} = -1$). The first ten triads evolve following the simplified Heider rules (see also Figure 1). Triads 11 and 12 have one intolerant element among the two elements with the same flag, while the element with the other flag is tolerant. Hence, there are two possible outcomes: the friend link is established between the two tolerant individuals (with probability q) or the friend link is established between the two individuals with the same flag (with probability $1 - q$). The remaining triads (thirteen to twenty) have only one path of evolution: a friend link between the two elements with the same flag and rival links between the elements with different flags.

having a different flag). As we pointed out in the introduction, throughout history, intolerance has had a great power of contagion due to the tolerance to intolerants. Parameter q will permit us to measure its effect on this spreading power. Note that the two possible balanced configurations both have the same number of negative links. That is, parameter q does not affect the degree of conflict at the triad level (the micro level), but, as will be shown later, parameter q exerts an important influence on the degree of conflict in the whole system (the macro level).

3. For the remaining eight triads (thirteen to twenty), the unique balanced configuration is a friend link between the two elements with the same flag and two rival links to the element with the other flag. The unique unbalanced configuration for these triads is the three-rival one.

The model therefore includes six parameters: the numbers of elements N_A and N_B with flag A and B, the numbers of intolerant elements N_{AI} and N_{BI} with flag A and B, the consensus probability p and the intolerance to intolerants probability q. Starting from a given initial condition, the numerical code randomly chose three elements $i, j,$ and k. If the triad formed by these elements is unbalanced, the rules in Figure 2 are applied to update the links (note that as a result of this update other triads containing elements (i, j) or (j, k) or (k, i) may switch from being balanced to being unbalanced or vice versa). New triads are then chosen and updated until no unbalanced triad remains. Depending on the parameter values and initial conditions, a large number of iterations may be needed to reach the final stable state.

Model Behavior

When the N elements in the system have the same flag, or all the elements are tolerant (i.e., all the triads are of type 1–10 as shown in Figure 2), the model is reduced to the model studied by Antal, Krapivsky, and Redner (2005). In this case, the triads follow the simplified Heider balance rules and the parameters are reduced to two: the size of the system N and the probability p of establishing an all-friends triad. As stated earlier, there are two stable collective solutions: one is the homogeneous solution with all $S_{i,j} = +1$ and the other is the polarization into two rival cliques where $S_{i,j} = +1$ for all members i and j within the same clique and $S_{i,j} = -1$ when members i and j belong to different cliques. The number of different balanced configurations is $1 + N/2$ (i.e., the number of members in the two cliques are: N and 0, or $N - 1$ and 1, . . . , or $N/2 + 1$ and $N/2 - 1$, or $N/2$ and $N/2$). These stable solutions cannot always be reached, however. Antal, Krapivsky, and Redner (2005) demonstrate that for $p < 1/2$, an infinite network starting from a random configuration quickly reaches a quasi-stationary dynamic state where unbalanced triads persist and that its density fluctuation is around a stationary value. For $p > 1/2$, the network evolves to the homogeneous situation in which no unfriendly relations remain. Although a finite network will sooner or later fall in an absorbing state for any value of p, the time to reach a final stable state increases exponentially (as $\exp(N^2)$) for $p < 1/2$.

When the elements in the system are distinguished with one of the two flags and some of the elements have the intolerant attribute, the number of parameters increases to six: N_A, N_B, N_{AI}, N_{BI}, p, and q. Introducing intolerants in the system has an important consequence in terms of segregation. Note that when there is at least

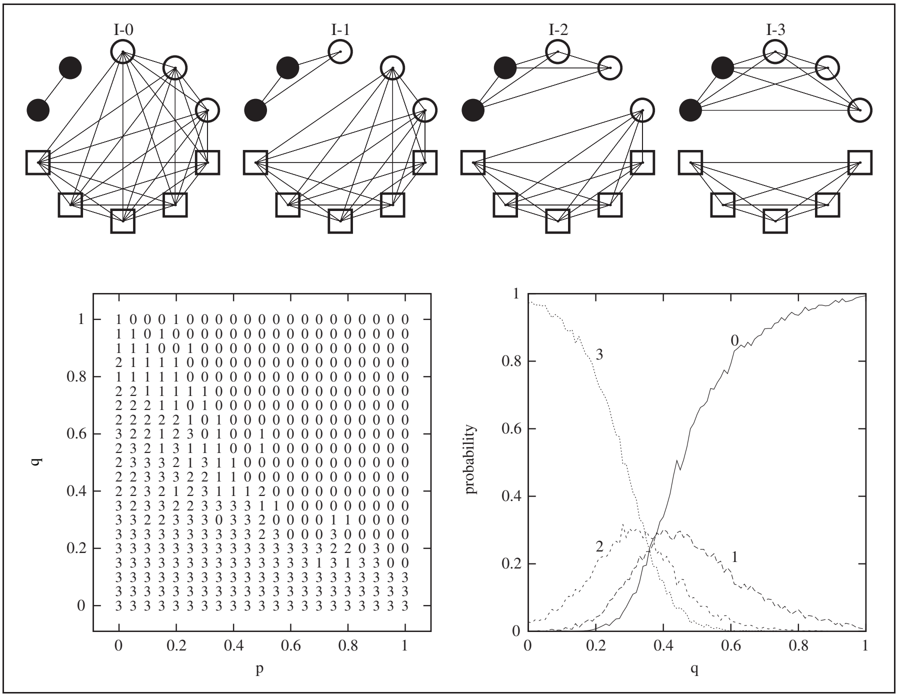

Figure 3. For $N_A = 5$, $N_{AI} = 2$, $N_B = 5$, and $N_{BI} = 0$: (Top) The four possible final balanced configurations. (Bottom-left) Final balanced configuration in the plane $p - q$; each label indicates the final configuration reached in a particular realization where the numbers 0, 1, 2, and 3 correspond to configurations I_0, I_1, I_2 and I_3, respectively. (Bottom-right) For 1,000 realizations, the bottom-right panel shows the probability of occurrence of each configuration as a function of q when the parameter p is fixed to 1/2.

one intolerant in each group, the unique possible balanced configuration is the segregation of the flags in two groups, hereafter Flag Segregated Configuration (FSC). This is a somewhat trivial result that involves a strong prediction since there is a unique solution in this case. Note that in comparison to the standard model, the presence of intolerants in both groups substantially reduces the number of balanced configurations from $1 + N/2$ to 1. However, if one of the groups does not possess intolerant elements, additional balanced configurations appear. These configurations range from the one in which the intolerants are isolated (hereafter configuration II; see the configuration at the left of the top panel in Figure 3) to FSC (see the right-hand configuration in Figure 3, top panel). Between the configurations II and FSC, there are $N_A - N_{AI} - 1$ configurations in which the intolerants form a clique with n of the tolerants having their same flag and the remaining tolerants of both flags form another clique (see Figure 3, top panel). Hereafter, we denote these stable states as I_n configurations where $0 \leq n \leq N_A - N_{AI}$. Note that I_0 corresponds to II and $I_{N_A - N_{AI}}$

to FSC. Note also that the number of negative links is maximum for the FSC and minimum for the II configuration.[3] If the degree of conflict is measured in terms of the number of negative links, then the FSC (having $N_A \times N_B$ negative links) is the balanced configuration with the maximum possible degree of conflict. The minimum degree of conflict corresponds to the II configuration (having $N_{AI} \times (N - N_{AI})$ negative links). A zero-conflict degree is only possible in the absence of intolerants, whereas in the case where intolerants are present in both groups, the unique balanced configuration FSC corresponds to the maximum degree of conflict.

As stated earlier, sooner or later a finite network freezes into one of the stable configurations. The occurrence of a given stable configuration depends on the parameter values, on the initial conditions, and on the sequence of link updates that are determined by a random number generator. The random number generator is used to choose the triad to be updated and to decide which link to update in the event that the update depends on parameter p or q. In what follows, all of our simulations start with all the links $S(i,j) = -1$ (i.e., all triads are unbalanced). This is an initial condition that is far from any of the balanced solutions of the system since all the triads are unbalanced. Additionally, negative links between the element types $AI - BT$, $BI - AT$, or $AI - BI$ are warranted from the beginning. The different realizations for the same set of parameter values and the same initial condition differ from each other on the seed used to start the random number generator that determines the sequence of triads to be updated.

Plane $p - q$ in the bottom-left panel of Figure 3 shows the stable configuration reached by a small system with two communities of the same size having some intolerants in one of the two communities. Each label in the $p - q$ plane indicates the configuration reached in a particular simulation, where the number labeled is the value of n corresponding to the configuration I_n reached in the simulation. Note that there is not a clear interface separating the occurrence of the various possible configurations in the $p - q$ plane because different sequences of triad updates can result in different configurations. For a given set of values (p, q), the probability of occurrence P_n of the stable configuration I_n can be inferred from the frequency of occurrence of the configuration in a large set of simulations. For the fixed value of parameter $p = 1/2$, the bottom-right panel in Figure 3 shows the probabilities $P_n (n = 0, 1, 2, 3)$ of the stable configurations as a function of the parameter q obtained from 1,000 different simulations for each value of q. Note that configurations II and FSC (configurations I_0 and I_3) have the highest frequencies. The configurations I_1 and I_2 occur with much less frequency and their probability of occurrence reaches a maximum when the probabilities are $P_0 \simeq P_3$. This suggests that, in the presence of noise (e.g., misleading flags), the intermediate configurations $(I_n, 0 < n < N_A - N_{AI})$ are not stable and can behave as transient quasi-stable states that finally converge to FSC or II configurations.

Figure 4 shows the probabilities P_n for a network that is twice as large as the one shown in Figure 3. Note that if the size of the network is increased by maintaining both the proportion in each flag and the fraction of intolerants, the transition from $P_{\mathrm{FSC}} \simeq 1$

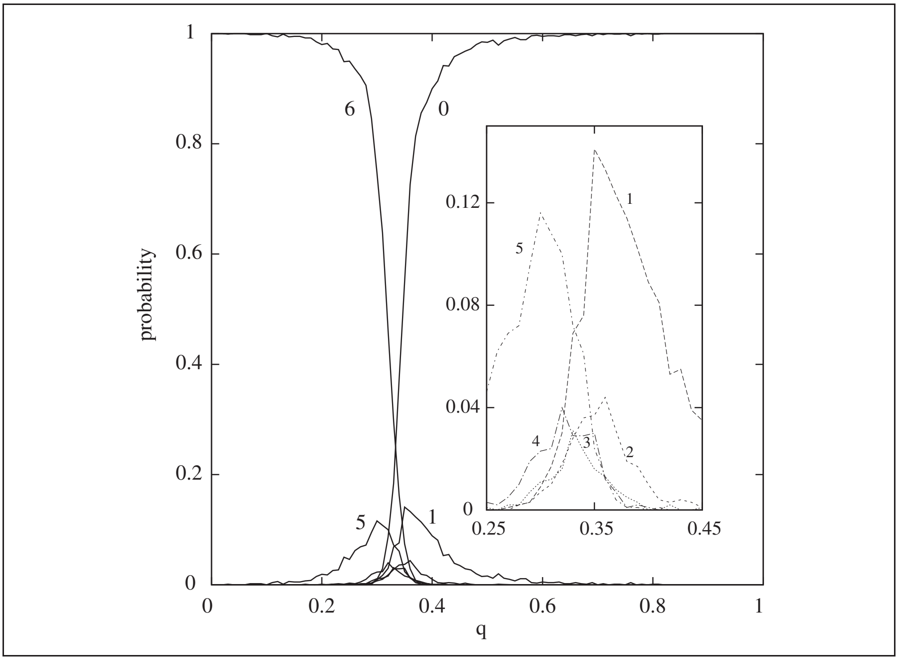

Figure 4. For a network with $N_A = 10$, $N_{AI} = 4$, $N_B = 10$, and $N_{BI} = 0$ (twice the case in Figure 3) and $p = 1/2$, the curves show the probability of occurrence P_0 to P_6 corresponding to the seven possible stable configurations as a function of q. The inset shows a zoom of the curves corresponding to the probabilities of occurrence of the intermediate configurations I_1 to I_5.

to $P_{\mathrm{II}} \simeq 1$ occurs at the same value of q as in the case shown in Figure 3, but the transition is much sharper. That is, as the network increases in size, the frequency of occurrence of the intermediate configurations decreases. As expected, FSC dominates at low values of q, while II dominates at high values of q. That is, there is a critical value $q_{\mathrm{cri}}(p)$ at which the degree of conflict changes abruptly from a high degree (when $q < q_{\mathrm{cri}}(p)$) to a low degree when $q > q_{\mathrm{cri}}(p)$. As noted by one of the referees, it is remarkable that a mechanism that does not impact the degree of conflict at the micro level appears to have a large impact on the degree of conflict at the macro level.

In the cases shown in the bottom-right panel of Figure 3 and Figure 4, the fraction of intolerants $F_I = N_{AI}/N_A$ was fixed to 2/5 and the consensus parameter to $p = 1/2$. In Figure 5, we now show how the probabilities $P_n(q)$ depend on F_I and p. Note that flag segregation is favored for low values of p or q, whereas high values of p or q promote the II. As expected, the probability of occurrence of flag segregation increases as the fraction F_I of intolerants increases. II is favored for low values of F_I. However, for low values of p and F_I (e.g., top-right panel in Figure 5), the intermediate configurations between FSC and II dominate for high values of q. Note that for a high enough

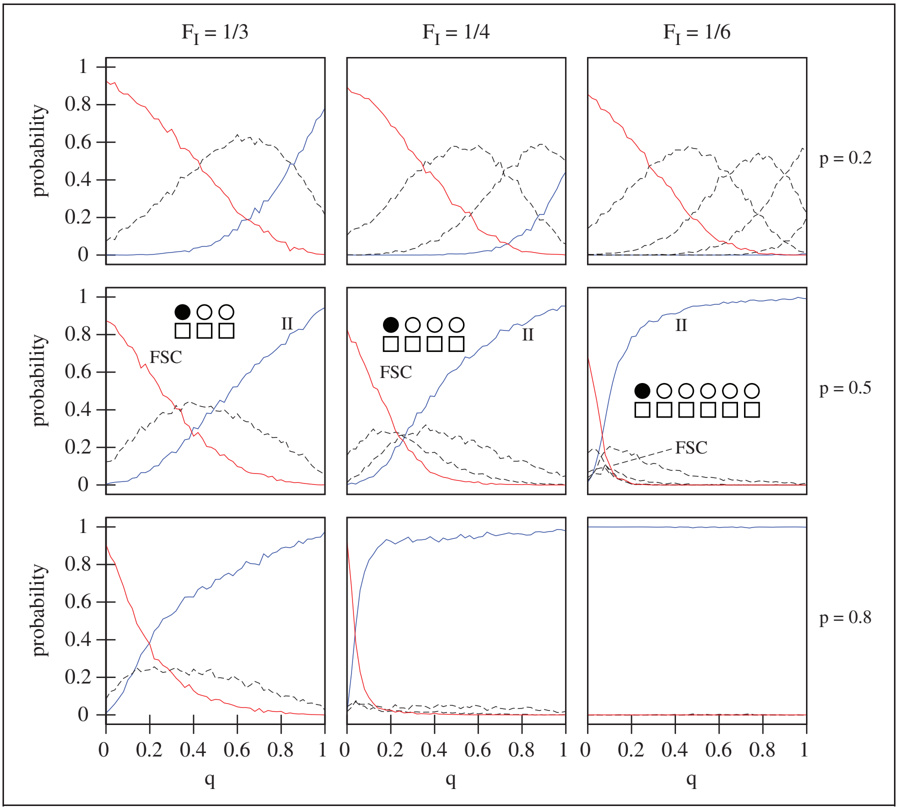

Figure 5. The probability of occurrence of the various possible balanced configurations as a function of the parameter *q*.

Note: From left to right, the three columns correspond to three different fractions of intolerant individuals: 1/3, 1/4, and 1/6, respectively, and three system sizes $N = 6$, 8, and 12, respectively. From top to bottom, the three files correspond to three values of parameter p: 0.2, 0.5, and 0.8, respectively. For each case (p, q, F_I), the probabilities are estimated from 1,000 realizations. The red curves give the probability of flag segregation P_{FSC}, while the blue curves give the probability of II P_{II}. The black curves correspond to balanced configurations between FSC and II. FSC = Flag Segregated Configuration; II = Isolation of Intolerants.

value of p and low enough values of F_I (e.g., bottom-right panel in Figure 5), intolerants are isolated for any value of q.

Larger Networks

In our model, the possible balanced configurations are known in advance, regardless of the size of the system. However, the number of link updates needed to reach one of these final configurations increases with the size N of the system and rapidly

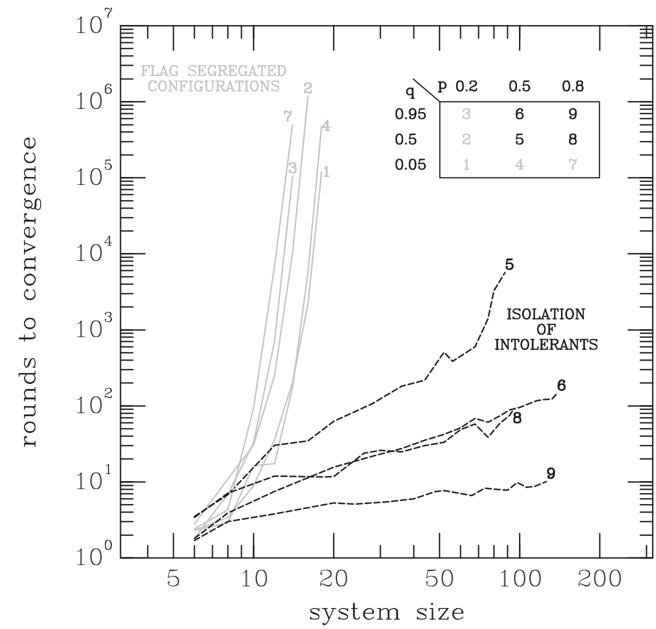

Figure 6. Convergence time (see text) as a function of the system size N for three values of the parameter p (0.2, 0.5, and 0.8) and three values of the parameter q (0.05, 0.5, and 0.95). *Note:* All simulations have $N_{AI} = 2$, $N_{BI} = 0$ and start with all links with $S = -1$ (i.e., all triads are unbalanced). The plotted convergence time is the average over ten different runs. The numbers in the table located near the upper-right corner and at the end of each curve indicate the values (p, q) used in each case. The cases that converge to FSC are shown as gray continuous curves and the cases that converge to II configurations are shown as black dashed curves. FSC = Flag Segregated Configuration; II = Isolation of Intolerants.

becomes computationally unaffordable. The convergence time (number of iterations to reach a balanced configuration) also depends on the initial condition, the parameters p and q, as well as on the number of intolerant elements in each group.

As a general rule, we observed that the number of updates to reach a balanced solution increases nonlineally with the size N of the system and depends on the type of the most probable final configuration, that is, FSC or II. Figure 6 shows the dependence of the convergence time as a function of the system size N for nine sets of the parameters (p, q). The time of convergence is measured in rounds units, where a round corresponds to $[N \times (N-1)]/2$ iterations, that is, a round is the minimum number of iterations needed to update all the links in the system once. The time of convergence for each set (N, p, q) in Figure 6 is the average time over ten runs. There is a clear difference in the evolution of the system when the conditions favor Flag Segregation or favor the II individuals. When the system parameters promote

Full Segregation (small values of parameters p and/or q; see also Figure 3 and the inset table in Figure 6; gray continuous curves in Figure 6), the convergence to the final balanced state requires large simulation times that are proportional to $\exp(N^2)$, and therefore only small systems ($N \sim 20$) can be simulated. When the system parameters promote the II (large values of parameters p and q; black-dashed curves in Figure 6), the convergence to the final balanced state requires shorter simulation times, and therefore larger systems up to $N \sim 150$ can be simulated. In these cases, the simulation times are proportional to N^{β}, with β increasing as p and q decrease.

As stated earlier, flag segregation is the unique possible balanced configuration when there are intolerants in both groups. Again, the time to reach this full segregated configuration grows superexponentially as $\exp(N^2)$ with size. The very long transient that precedes the establishment of the FSC can be associated to an endemic quasi-stable state. The existence of these long-standing, partially balanced configurations can be evoked to explain why complete segregation is rarely observed in real systems. The dynamical properties of this kind of long transients have been studied recently by Ludwig and Abell (2007) and Abell and Ludwig (2009) for a system that shares some properties with our model, but that consider the evolution of incomplete networks with signed links that can be created and deleted between identical nodes.

Larger systems cannot be simulated unless the dynamical rules that drive the evolution of our model are modified to accelerate convergence. A possible modification that was considered by Antal, Krapivsky, and Redner (2005) consists in constricting the dynamics by rejecting any link update that increases the number of unbalanced triads, which the authors refer to using the acronym constrained triad dynamics (CTD). With CTD, the system is trapped much faster in a steady configuration that can possess unbalanced triads (a jammed state in the terminology of Antal, Krapivsky, and Redner [2005]). Although we do not consider pure CTD in the present work, in the Appendix we have explored a modification that consists in applying CTD most of the time, but sporadically allowing a "re-heating."

Even with the modified model described in the Appendix, the currently available computing capacity only permits simulating relatively small networks. However, these small networks are sufficient to simulate communities whose members are able to maintain a continuous relationship with each other (Dunbar's number; Dunbar 1992). In large communities, the individuals maintain steady relations with a small fraction of the community members. If this limitation is included in a model similar to the one considered here, the maximum size of the network that can be simulated is expected to increase substantially. It remains to be proven that in partially connected networks the degree of segregation and conflict decrease as the parameters p and q increase, but a priori we do not see any reason that could produce a different tendency.

Related Works and Possible Refinements

Among the works that explicitly link homophily and tolerance–intolerance, Gracia-Lázaro, Floría, and Moreno (2011) incorporated a tolerance parameter Z in

Axelrod's (1997) model of cultural dissemination. This parameter denotes a cultural trait that controls the likelihood of rewiring, where lower values of Z imply less tolerant attitudes and a higher likelihood of rewiring. The authors of this interesting work show that for some intermediate values of Z, cultural globalization is disfavored in comparison to Axelrod's model where rewiring is not considered. Note that in this model all elements have the same steady value of Z and therefore there is no way to analyze the influence of tolerants over intolerants. Even if the Z parameter were allowed to be inhomogeneous, the T-dilemma is not present because the links established by the tolerants are not directly influenced by the presence of intolerants.

Traulsen and Claussen (2004) propose another model that explicitly considers the effect of intolerance on segregation. In their model, a cooperative game is played by similar and dissimilar players in a two-dimensional lattice. The authors show that tolerant strategies can only survive with sufficient support from surrounding players. Tolerance is understood here as an "all-or-nothing" decision because tolerant players ($T = 1$) cooperate with all the other players and intolerant players ($T = 0$) cooperate only with players of the same tag. The evolution of the player's strategies drives the system to strongly segregated states where the majority of players are intolerant in equilibrium. However, if stochastic mutations are included in the model, population dynamics work against this equilibrium (Traulsen and Schuster 2003; Traulsen and Claussen 2004). Note again that the T-dilemma is not present in this model since the relations (in this case cooperation) established by a tolerant player are not directly influenced by the presence of an intolerant player.

Several recent works have considered variations of triad dynamics to study the structural balance of discrete systems. For example, van de Rijt (2011) considers a modification of the balance principle proposed by Davis (1967), where the triad consisting of three negative relations is now also balanced.

Abell and Ludwig (2009) consider the evolution of signed relations in incomplete linked networks of up to 100 nodes. The evolution is driven by the creation and the deletion of links instead of the change of sign used in our model. These two processes have not been considered in our fully connected model, but certainly have a relevant role in real systems. Other classes of relation between the network nodes have been considered, such as neutral, null, or awareness relations (Montgomery 2009).

Marvel et al. (2011) approach the problem of structural balance with a description in which the time and the intensity of the links are continuous. In contrast, in our model, the possible values adopted by a link are $+1$ or -1, and the time is measured in discrete iteration units. In the continuous approach, the evolution of the intensity x_{ij} of the link between two nodes i, j is given by the ordinary differential equation $\frac{dx_{ij}}{dt} = \sum_k x_{ik} \times x_{kj}$, where k includes all other nodes in the network. As in the discrete model, this system of ordinary differential equations evolves toward a situation in which all relationships become friendly or two antagonist cliques emerge. An important difference with the discrete model studied here is that the continuous model is completely deterministic. The size of systems that can be simulated in the discrete and continuous models is similar.

None of the preceding models includes intolerant individuals and the dynamics associated to the q parameter. The combination of some of the processes considered in these models with the dynamics associated to the q parameter could allow for an analysis of the process of segregation in more realistic contexts. However, the basic features discovered in the simplest model considered here serves as a valuable guide for these explorations. More realistic models could also consider heterogeneous systems in which the individuals possess different values of the parameters p and q. Moreover, the parameters p and q can evolve in time due to feedback processes. Finally, in a realistic model, the attitude and/or appearance of the individuals can change in response to environmental conditions. These feedback processes have the potential to accelerate the convergence toward balanced configurations, or to give stability to unbalanced configurations.

Discussion

Intolerance is one of the most widespread and persistent social phenomena with the greatest capacity to cause segregation and conflict. Intolerance persists even in the most democratic countries with a history of tolerance (McGhee 2005). Moreover, intolerance is not only a ubiquitous phenomenon but has a great power of contagion due largely to homophilic pressures. Tolerant persons could then be trapped in the dilemma of tolerating in-group intolerants, giving in to homophilic pressures, or not tolerating them. In spite of the social and historical pervasiveness of these phenomena, the dynamics that generate either macro-level segregation into cliques or its integration by excluding intolerants are not well known. Some elements of our social balanced network model can help better understand these processes.

The proposed model consists in a finite network of fully connected elements that can be of four types depending on the element's flag (A or B) and the element's attitude (tolerant or intolerant). The links between these elements evolve toward a balanced (stable) configuration following a set of simple rules (the micro-level mechanisms). The model behavior depends only on six parameters (the number of tolerant and intolerant individuals in each flag (four) plus the consensus parameter p and the T-dilemma parameter q). The analysis of the model has focused on determining the necessary conditions (parameter values) to reach various types of balanced configurations that range from all-friend to segregated societies. The possible equilibrium configurations to which our model can converge are achieved with probabilities that depend on the parameters p and q.

As in previous studies (Antal, Krapivsky, and Redner 2005), parameter p determines the tendency (probability) of individuals to establish positive links that promote the establishment of balanced three-friend triads. This parameter has important effects on the segregation since segregation requires negative links between the two sides and small values of p favor the establishment of triads in conflicting equilibrium. In contrast, high values of p favor the establishment of three-friend triads. In the context of our model, high values of p favor the establishment of positive links between tolerant individuals of both groups, which can unbalance triads with intolerant individuals and result in the isolation of these individuals.

Moreover, parameter q was introduced in our model to handle a crucial aspect of intolerance which we have called the *T-dilemma*, the dilemma faced by a tolerant person who has to choose between establishing a positive relation with a tolerant individual of a dissimilar group or establishing a positive relation with an in-group intolerant member. The q parameter measures the prevalence of intolerance to intolerants over homophily on triads with individuals of both flags, which include an intolerant individual. In these triads, a three-friends balance is excluded and therefore the only possible balanced configuration is the conflicting equilibrium. The parameter q has an influence on which pair of individuals will establish the positive link, either between two with the same flag (homophily) or between two tolerant individuals with different flags (rejection of intolerance). For $q = 1$, the tolerant individual always rejects the intolerant in-group member, but for smaller values, q indicates the likelihood that the rejection of intolerance will prevail over homophily.

The q parameter—and hence the T-dilemma—has shown to have an important influence at the macro level. At the micro level, however, this parameter has no effect whatsoever on the degree of conflict of the conflictive triad. It is worth noting that the complex dynamics that lead to the segregation of a society in two cliques or to the exclusion of intolerants at the macro level do not stem from a "complex dynamics at the micro base" in our model (Hedström and Bearman 2009, 13), but from very simple ones. The unexpected macro result shows the fertility and analytical usefulness of social balanced models, indicating that caution must be taken in generalizing results from micro, face-to-face relationships to macro-level states. This is a possible reason why the Contact hypothesis, even if it were true, could be insufficient for a society to avoid intolerant segregation at the macro level.

A strong prediction in the context of our model is that when there are intolerant individuals on both sides, the only possible balanced configuration is the total segregation of the two flags. In our model, the triads cannot reach a three-friends balance when two tolerants with different flags interact with an intolerant. It is sufficient to have a minority of intolerants in both groups to establish a large number of triads in conflictive equilibrium that finally produce the complete segregation of the two communities with different flags. This is a suggestive property of the model since it is a historically observed fact that small, active groups of intolerant individuals in each side suffice to destabilize many social relationships and, ultimately, the entire society. Complete segregation is rarely observed in real systems, but in our model the establishment of the FSC in large systems is preceded by very long transient states that can be associated to endemic quasi-stable states characterized by high degrees of conflict.

When only one of the groups contains intolerants, new balanced configurations can occur, and the number of three-friend triads is, in general, larger than in the case where both groups contain intolerants. If, in addition, $q \simeq 1$ (the tolerant persons do not tolerate the intolerants), the intolerants are isolated (II configuration), and the number of positive links between members of both flags reach the maximum possible value. Hence, there is less segregation because it only affects the intolerant individuals. If instead $q \simeq 0$ (the tolerants tolerate the in-group intolerants and establish positive links with them), the tolerant individuals of one of the groups act as intolerants whenever they find themselves in the T-dilemma and the society is segregated into the two flags (FSC). Depending on the parameters p and q, the system tends to reach one of the $N_A - N_{AI} + 1$ possible configurations, but are more likely to reach the two extreme configurations II and FSC. As the number of individuals in the system increases, the intermediate stable solutions (I_1 to $I_{NA-NAI-1}$) tend to be increasingly rare, and the time required to reach the extreme solutions FSC and II also increases in a substantial manner.

It is worth noting that different configurations can be achieved even for the same parameter values due to chance events involved in the evolution of links in the unbalanced triads. This is typical of social systems where random or environmental events, which although often seemingly insignificant, are essential to rebuild the thread of history.

The final $N_A - N_{AI} + 1$ possible configurations are known in advance, but the number of link updates needed to reach one of these final configurations increases with the size N of the system and rapidly becomes computationally unaffordable. When the system parameters promote full segregation (small values of parameters p and/or q, the convergence to the final balanced state requires large simulation times and only small systems ($N \sim 20$) can be simulated. When the system parameters promote II (large values of parameters p and q) the convergence to the final balanced state requires shorter simulation times, and systems up to $N \sim 150$ can be simulated. It is common to find that real social networks are only partially balanced and show intermediate degrees of segregation between FSC and II. However, these situations do not necessarily contradict the results obtained here as the observed configurations could correspond to the transient states in our model simulations. If, in addition, we consider that real systems are open (with individuals entering and exiting them), that individuals make mistakes when assessing the intentions of others, and that real systems are subject to external influences, then these partially balanced configurations may be endemic (when the production of balanced social relations is compensated by the creation of unbalanced relations due to the changes mentioned earlier). Antal, Krapivsky, and Redner (2005) described another mechanism that can aid in explaining the fact that real social networks are not fully balanced according to the criterion of Cartwright and Harary. This mechanism assumes that an individual will only modify an unsatisfactory relationship with another individual provided that such a modification does not create more tension with other individuals, that is, when there are more balanced than unbalanced triads. The model can also be modified to take into account that there is a relatively small limit on the number of social relations that individuals can engage in (Dunbar 1992). For systems with a much larger number of individuals than Dunbar's number, the network of interactions is sparse and many triads cannot be closed. In addition to positive or negative links, individuals could have the option of disconnecting, and the relation between two individuals may even be asymmetric. These networks may display features that are not present in the fully connected network considered here, but one would expect that the main properties of segregation of our model persist after including these refinements.

The results from an artificial society such as the one presented here cannot usually be directly extrapolated to concrete real situations. Nonetheless, the results seem to point to the idea that the zero-tolerance strategy (i.e., $q \simeq 1$) is, as assumed by many public campaigns, a very effective measure against intolerance and segregation. In addition, democratic education promoting the resolution of conflicts through negotiation is a complementary strategy for combating segregation, which in our model is equivalent to increasing parameter p. Another suggestive result refers to the impossibility of reducing the conflict in bilateral disputes while intolerant subgroups remain active in both groups. The model suggests that an initial step toward the resolution of bilateral disputes requires that some circumstance demobilize the more radical members (intolerants) of one of the factions or at least that they become invisible or are considered a third group. This sets the stage for the second group to isolate their intolerant members and solve the conflict.

Appendix

CDT Model with Sporadic Reheating

To explore the effect of the mechanisms associated to parameters p and q in larger systems, we have modified the dynamical rules that drive the evolution of our model for the purpose of accelerating the convergence to the final balanced configurations. A possible modification that was considered by Antal, Krapivsky, and Redner (2005) consists in constricting the dynamics by rejecting any link update that increases the number of unbalanced triads, which they refer to by the acronym constrained triad dynamics (CTD). With CTD, the system is trapped much faster in a steady configuration that can possess unbalanced triads (a jammed state in the terminology of Antal, Krapivsky, and Redner 2005). We do not consider pure CTD in the present work because most of the final steady configurations that are reached with this dynamic do not coincide with the final balanced configurations I_n, $0 \leq n \leq N_{AT}$. To eliminate the nonbalanced final configurations, we have explored a modification that consists in applying CTD most of the time, but sporadically allowing a "re-heating" (hereafter referred as CDT + RH). That is, links are updated only if the total number of unbalanced triads do not increase (CTD), but sporadically this constraint is released. This sporadic reheating prevents the system from being trapped in nontotally balanced configurations. However, in addition to accelerating the convergence, the model with CDT + RH favors the occurrence of intermediate configurations I_n in between FSC ($n = N_{AT}$) and II ($n = 0$). It could be interesting to analyze a model with CDT + RH in more detail since it can be associated to the

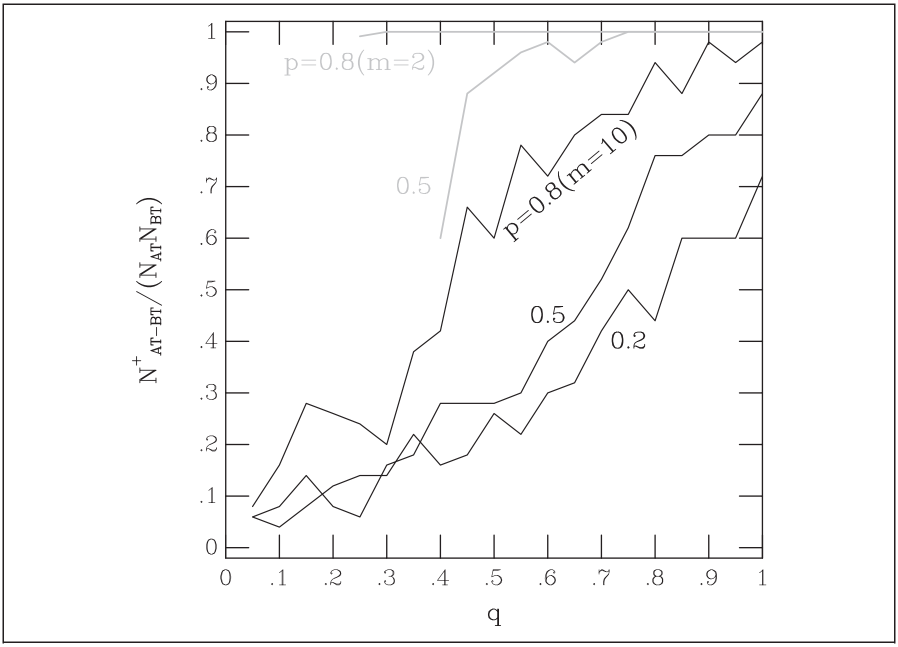

Figure A1. Level of Integration as a function of the parameter q for three values of the parameter p (0.2, 0.5, and 0.8).
Note: The integration is measured as the ratio of the number of positive links between the tolerant individuals in both communities to the maximum possible value $N_{AT} \times N_{BT}$. All simulations have $N_{AI} = 10$, $N_{AT} = 50$, $N_{BT} = 60$, and $N_{BI} = 0$, and start with all links with $S = -1$ (i.e., all triads are unbalanced). The update dynamics in these simulations allow link changes that result in increased conflict in the system only each m iterations (see text). The black curves correspond to simulations with $m = 10$ and the gray curves to simulations with $m = 2$. The results shown correspond to the final balanced configurations that are reached in less than 100 rounds (in this case, each round contain 7,140 iterations).

tendency of people to avoid actions that increase the level of conflict unless being trapped for some time in an uncomfortable situation. We introduce CDT + RH in the update dynamics by allowing link changes that result in increased conflict in the system only each m iterations. However, for large systems, it should be borne in mind that, as noted by Abell and Ludwig (2009), CTD dynamics are difficult to justify because the calculation capabilities of those involved would be brought seriously into question.

Figure A1 shows the results for a system that is ten times larger ($N = 120$) than the case $F_I = 1/6$ and $N = 12$ shown in the panels in the right column of Figure 5. The update dynamic in these simulations is CDT + RH with $m = 2$ or $m = 10$. Note that this modified model reduces to the nonmodified model when $m = 1$.

To summarize the results, we calculate the "Integration Level" between the two communities, or equivalently, the degree of isolation of the intolerant individuals, which is defined as the ratio $\frac{N^+_{AT-BT}}{N_{AT} \times N_{BT}}$, where the numerator is the number of positive links among the communities A and B, and the denominator is the maximum possible value of N^+_{AT-BT}.

The integration level is shown in Figure A1 as a function of the parameter q for three values of the parameter p (0.2, 0.5, and 0.8). The results shown correspond to final balanced configurations that are reached in less than 100 rounds. Due to computational limitations, only one run was performed for each set (m, p, q), which is why the curves are not smooth. Note that in less than 100 rounds, the $m = 10$ case converges for all sets (p, q), the case with $m = 2$ only converges for a fraction of the sets (p, q), and the case with $m = 1$ (i.e., the nonmodified model) only converges in less than 100 rounds for values of p and q close to one as shown in Figure 6. Note also that the limited results for the case $m = 2$ (the closest case to the nonmodified model) coincide with the results for the ten times smaller system shown in the right column of Figure 5. As m increases, the intermediate configurations between FSC and II become more common. Nevertheless, these results indicate that the general properties of the nonmodified model ($m = 1$) for small networks remain valid for larger systems with $m > 1$, that is, the degree of segregation and conflict between the two communities decreases as the parameters p and q increase.

Acknowledgments

We wish to thank the two anonymous referees for many helpful comments that significantly improved the article.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: Antonio Parravano was supported as guest researcher by the Instituto de Estudios Sociales Avanzados (IESA-CSIC), Córdoba, Spain.

Notes

1. Fully connected networks are appropriate for describing social interactions in relatively small groups, that is, groups smaller than the average number of social relations a person can maintain (this number is in the range of 100 to 150 and is known as the *Dunbar's number*; Dunbar 1992, 1998).
2. The number of combinations of n distinct elements in groups of r elements allowing repetitions is $(n + r - 1)!/(r!(n - 1)!) = 20$ for $n = 4$ and $r = 3$.
3. The number of negative links in a I_n configuration is $N^- = (N_{AI} + n)(N - (N_{AI} + n))$.

References

Abdou, M., and N. Gilbert. 2009. "Modelling the Emergence and Dynamic of Social and Workplace Segregation." *Mind and Society* 8:173-91.

Abell, P., and M. Ludwig 2009. "Structural Balance: A Dynamic Perspective." *The Journal of Mathematical Sociology* 33 (2): 129, 155.

Antal, T., P. L. Krapivsky, and S. Redner. 2005. "Dynamics of Social Balance Networks." *Physical Review E*, 72:036121.

Axelrod, R. 1997. "The Dissemination of Culture: A Model with Local Convergence and Global Polarization." *Journal of Conflict Resolution* 41:203-26.

Cartwright, D., and F. Harary. 1956. "A Generalization of Heider's Theory." *Psychological Review* 63:277-92.

Castellano, Claudio, M. Marsilli, and A. Vespignani. 2000. "Nonequilibrium Phase Transition in a Model for Social Influence." *Physical Review Letters* 85:3536-39.

Centola, D., J. C. González-Avella, V. M. Eguíluz, and M. San Miguel. 2007. "Homophily, Cultural Drift, and the Co-evolution of Cultural Groups." *Journal of Conflict Resolution* 51:905-29.

Davis, James A. 1967. "Clustering and Structural Balance in Graphs." *Human Relations* 20: 181-87.

Dixon, J., K. Durrheim, and C. Tredoux. 2005. "Beyond the Optimal Contact Strategy: A Reality Check for the Contact Hypothesis." *American Psychologist* 60 (7): 697-711.

Dunbar, R. I. M. 1992. "Neocortex Size as a Constraint on Group Size in Primates." *Journal of Human Evolution* 22:469.

Dunbar, R. I. M. 1998. "The Social Brain Hypothesis." *Evolutionary Anthropology* 6:178-90.

Gawroński, P., P. Gronek, and K. Kułakowski 2005. "The Heider Balance and Social Distance." *Acta Physica Polonica B* 36:2549.

Gracia-Lázaro, C., L. M. Floría, and Y. Moreno. 2011. "Selective Advantage of Tolerant Cultural Traits in the Axelrod-Schelling Model." *Physical Review E* 83:056103.

Gracia-Lázaro, C., L. F. Lafuerza, L. M. Floría, and Y. Moreno. 2009. "Residential Segregation and Cultural Dissemination: An Axelrod-Schelling Model." *Physical Review E* 80:046123.

Hedström, P., and P. Bearman. 2009. "What Is Analytical Sociology All About? An Introductory Essay." In P. Hedström and P. Bearman (eds.), *Handbook of Analytical Sociology*, 3-24. Oxford, UK: Oxford University Press.

Heider, F. 1946. "Attitudes and Cognitive Organization." *Journal of Psychology* 21:107-12.

Heider, F. 1958. *The Psychology of Interpersonal Relations*. New York: John Wiley.

Hummon, N. P., and P. Doreian. 2003. "Some Dynamics of Social Balance Processes: Bringing Heider Back into Balance Theory." *Social Networks* 25:17-49.

Kułakowski, K., P. Gawroński, and P. Gronek. 2005. "The Heider Balance—A Continuous Approach." *The International Journal of Modern Physics C* 16:707.

Ludwig, M., and P. Abell. 2007. "An Evolutionary Model of Social Networks." *The European Physical Journal B* 58:97.

Marvel, S., J. Jon Kleinbergb, D. Robert, R. D. Kleinberg, and S. Strogatz. 2011. "Continuous-Time Model of Structural Balance." *Proceedings of the National Academy of Sciences* 108:1771-76.

Marvel, S., S. Strogatz, and J. Kleinberg. 2009. “Energy Landscape of Social Balance.” *Physical Review Letters*, 103:198701.

McGhee, D. 2005. *Intolerant Britain? Hate, Citizenship & Difference*. Maidenhead, UK: Open University Press.

McPherson, M., L. Smith-Lovin, and J. M. Cook. 2001. “Birds of a Feather: Homophily in Social Networks.” *Annual Review of Sociology* 27:415-44.

Montgomery, J. D. 2009. “Balance Theory with Incomplete Awareness.” *The Journal of Mathematical Sociology* 33:69-96.

Noël, L. 1994. *Intolerance: A General Survey*. Montreal, Canada: McGill-Queen’s Press.

Parravano, A., H. Rivera-Ramírez, and M. G. Cosenza. 2007. “Intracultural Diversity in a Model of Social Dynamics.” *Physica A*, 379:241-49.

Schelling, T. C. 1971. “Dynamic Models of Segregation.” *Journal of Mathematical Sociology* 1:143-86.

Sigelman, L., and S. Welch. 1993. “The Contact Hypothesis Revisited: Black-White Interaction and Positive Racial Attitudes.” *Social Forces* 71:781-95.

Traulsen, A., and J. C. Claussen. 2004. “Similarity-based Cooperation and Spatial Segregation.” *Physical Review E* 70:046128.

Traulsen, A., and H. G. Schuster. 2003. “A Minimal Model for Tag-based Cooperation.” *Physical Review E* 68:046129.

van de Rijt, A. 2011. “The Micro-Macro Link for the Theory of Structural Balance.” *The Journal of Mathematical Sociology* 35:94-113.

Zachary, W. W. 1977. “An Information Flow Model for Conflict and Fission in Small Groups.” *Journal of Anthropological Research* 33:452-73.