Hierarchical structure induced by external input in coevolving dynamical networks

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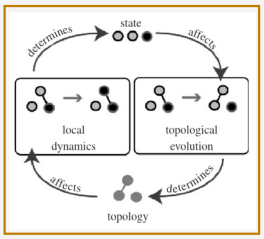
We investigate a coevolving network of coupled maps subject to a forcing as a simple model for an adaptive system of neurons receiving an external stimulus. Coevolution means that the dynamics of the maps modifies the connectivity structure of the underlying network, and that this structure in turn affects the dynamics, both varying in time. The local dynamics is chaotic and described by the circle map. The influence of the intensity of the external stimulus on the dynamical collective behavior and the topological properties of the network are studied. It is shown that there are ranges of the intensity of the stimulus for which hierarchical structures arise in the network. In addition, it is found that the stimulus induces more structure when the states of the maps are less synchronized. Our work illustrates the potential of coupled map models for studying coevolution in complex systems.

Introduction



The study of complex networks has become a new paradigm for the understanding of complex systems[1]. Some of these complex networks are adaptive, which means that the dynamics of the nodes modifies the connectivity structure of the underlying network, and that this structure in turn affects the dynamics, both varying in time [2].

In recent studies, based on both theoretical and experimental results, fast synaptic change has been proposed as an important process in the temporal coding of information[3]. In order to determine whether genetic information is necessary for the formation of hierarchical structure, it is important to study if such structure can be formed spontaneously through dynamics alone.

In this work, we investigate a coevolving network of coupled maps subject to a forcing as a simple model of plastic system of neurons subject to an external input.

The Model

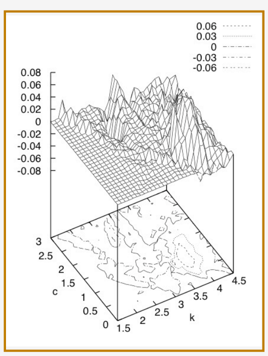
We consider a globally coupled map network model introduced by Ito and Kaneko [4], that has the following properties:

- Global connections between nonlinear units that can exhibit chaotic and other dynamic behavior. The local dynamics is described by the circle map.
- Plastic change of the connections between units that depends not only on the states of the two units to which they correspond but also globally on all the other units.
- An external input applied to one unit to change their states.

$$x^i_{n+1} = x^i_n + \Omega + \frac{k}{2\pi}\sin(2\pi x^i_n) + \frac{c}{2\pi}\sum_{j=1}^{N}\epsilon^{ij}_n \sin(2\pi x^i_n) + I^i$$

$$\epsilon^{ij}_{n+1} = \frac{[1+\delta\cos 2\pi(x^i_n - x^j_n)]\epsilon^{ij}_n}{\sum_{j=1}^{N}[1+\delta\cos 2\pi(x^i_n - x^j_n)]\epsilon^{ij}_n}$$

$$\begin{aligned} x^i_n &= \text{state of unit } i \text{ at discrete time } n,\ i=1,\dots,N \\ \epsilon^{ij}_n &= \text{strength of the coupling from } i \text{ to } j \\ \delta(0\le\delta\le 1) &= \text{plasticity parameter; } c = \text{global coupling strength} \\ I^i = I\delta_{ij} &= \text{external input},\ j=1 \end{aligned}$$



$$A(I=0) - A(I=0.2)$$

$$A = \frac{1}{(N-1)^2}\frac{1}{\tau_m}\sum_{i\neq j}\sum_{n=\tau_t}^{\tau_t+\tau_m}|\epsilon^{ij}_n - \epsilon^{ij}_{n-1}|$$

Figure 2.. Difference between the variations of ϵ_{ij} for the cases with and without stimuli. A strong input dependency is observed in the region 3.5 < *k* < 4.2 and 0.2 < *c* < 2.

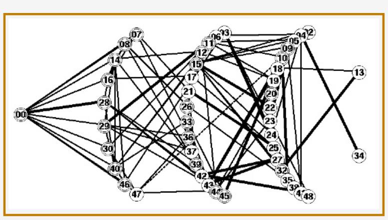


Figure 3. Hierarchical layer structure induced on the network by the external input applied on one unit.

Structure formation in adaptive networks with variable input

We study the influence of the external input ***I*** in the formation of the structure in the network [5].

The topological structure of the network can be characterized by the clustering coefficient:

$$C = \langle C_i\rangle_i = \left\langle \frac{n_i}{\frac{1}{2}k_i(k_i-1)}\right\rangle_i = \frac{1}{N}\sum_{i=1}^{N}\frac{n_i}{\frac{1}{2}k_i(k_i-1)}$$

$$\begin{aligned} k_i &= \text{degree of node } i \\ n_i &= \text{total connections between closest neighbors of i} \\ N &= \text{total number of nodes} \end{aligned}$$

The synchronization of the network can be characterized by the instantaneous standard deviation:

$$\sigma_n = \left[\frac{1}{N}\sum_{i=1}^{N}(x^i_n - \langle x_n\rangle)^2\right]^{1/2} \qquad \langle x_n\rangle = \text{instantaneous average of } x^i_n, \forall\, i$$

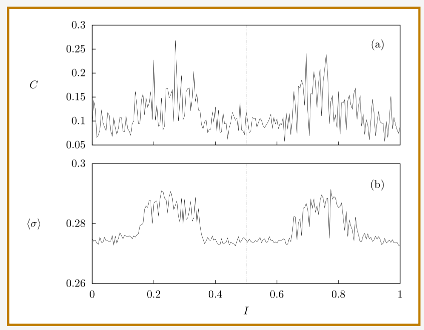


Figure 3. (a) Clustering coefficient vs. Input ***I***. (b) Asymptotic time average of the standard deviation vs. Input ***I***. Fixed parameters $\delta = 0.1,\ \Omega = 0,\ N = 50$

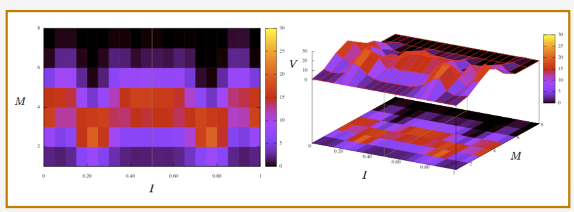


Figure 4. Number of nodes V appearing in different layers M, as a function of I, with $\delta = 0.1,\ \Omega = 0,\ N = 50$.

Conclusions

- There is a range of the intensity of stimulus ***I*** for which hierarchical structure emerges in the network.
- The external stimulus induces more structure in the adaptive network when the states of the maps are less synchronized. This result has important implications in the processing of information in the nervous system. In particular recent experimental studies show that the synchronization in some areas of the brain are correlated with the functioning long term memory [6].

REFERENCES

[1] S.Boccaletti, *et al.*, *Physics Reports*, **424**,175-308 **(2006).**

[2] T. Gross, B. Blasius, *J. R. Soc. Interface* **5**: 259-271 **(2008).**

[3] M. V. Tsodyks, H. Markram. Proc. Natl. Acad. Sci. USA, **94**, 719-723 **(1997).**

[4] J. Ito, K. Kaneko, Neural Networks, **13,** 275 **(2000).**

[5] V. J. Marquez, M. G. Cosenza, K. Tucci, *Revista Ciencia e Ingenieria*, **32,** 101-106 **(2011).**

[6] T. Zhuang, *et al.*, Computer and Information Science, **2**, 109-114, **(2009).**