The concept of transfer entropy is best understood within the context of information theory. Information is any event that can change the state of a system. The general idea is to optimally encode messages such that they can be transmitted more quickly. The quantity that can be calculated from a specific sequence of transmitted symbols[1]. Based on the measure of information transfer between two variables, we define the self-information transfer for time series of maps. This quantity can characterize periodic orbits.

### Time-Series for Maps

We consider the time series generated by a map, such as the logistic function.

#### Logistic Map:

\[ x_{n+1} = rx_n(1 - x_n) \]  (1)

We consider a time series for a given parameter value, and the one-step delayed corresponding time series:

\[ I = \{x_0, x_1, x_2, \ldots, x_{n-1} \} \]
\[ f = \{x_1, x_2, x_3, \ldots, x_n \} \]  (2)

The time series can be transformed into symbolic dynamics:

\[ x_n \leq 0.5 \rightarrow 0 \quad \text{and} \quad x_n \geq 0.5 \rightarrow 1 \]  (3)

### Time-Series for Maps

The information flow between two variables \( y \) and \( x \) can be characterized by the measure of information transfer introduced by Shreiber [1]

#### Transfer Information:

\[ T_{y \rightarrow x} = \sum_{x_{n+1}, x_n, y_n} p(x_{n+1}, x_n, y_n) \log \left( \frac{p(x_{n+1}, x_n) p(x_n)}{p(x_n, y_n) p(x_{n+1}, x_n)} \right) \]  (4)

We define the Self-Information Transfer:

#### Self-Information Transfer:

\[ T_{x_{n+1} \rightarrow x_n} = \sum_{x_{n+1}, x_n, x_{n-1}} p(x_{n+1}, x_n, x_{n-1}) \log \left( \frac{p(x_{n+1}, x_n, x_{n-1}) p(x_n)}{p(x_n, x_{n-1}) p(x_{n+1}, x_n)} \right) \]  (5)

### Results

- **Fig 1.** Bifurcation diagram for the logistic map.
- **Fig 2.** Lyapunov exponent for the logistic map vs. \( R \).
- **Fig 3.** Averaged Self-Information Transfer \( T_{x_{n+1} \rightarrow x_n} \) as a function of \( R \).
- **Fig 4.** Averaged Self-Information Transfer \( T_{x_n \rightarrow x_{n-1}} \) as a function of \( R \).

### Conclusions

- Self-Information Transfer is maximum when the system is in a periodic orbit.

### References
