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Phase ordering induced by defects in chaotic bistable media

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Abstract. The phase ordering dynamics of coupled chaotic bistable maps on lattices with defects is investigated. The statistical properties of the system are characterized by means of the average normalized size of spatial domains of equivalent spin variables that define the phases. It is found that spatial defects can induce the formation of domains in bistable spatiotemporal systems. The minimum distance between defects acts as parameter for a transition from a homogeneous state to a heterogeneous regime where two phases coexist The critical exponent of this transition also exhibits a transition when the coupling is increased, indicating the presence of a new class of domain where both phases coexist forming a chessboard pattern.

Coupled map lattices constitute fruitful and computationally efficient models for the study of a variety of dynamical processes in spatially distributed systems [1]. The discrete-space character of coupled map systems makes them specially appropriate for the investigation of spatiotemporal dynamics on nonuniform and on complex networks [2–5].

There has been recent interest in the study of the phase-ordering properties of systems of coupled chaotic maps and their relationship with Ising models in statistical physics [6–12]. These works have generally assumed the phase competition dynamics taking place on a uniform space; however, in many physical situations the medium that supports the dynamics can be nonuniform due to the intrinsic heterogeneous nature of the substratum such as porous or fractured media, or it may arise from random fluctuations in the medium. This paper investigates the process of phase ordering in coupled chaotic maps on a lattice with defects as a model for studying this phenomenon on nonuniform media.

We consider a system of coupled maps defined on a two-dimensional square lattice of size $N = L \times L$ with periodic boundary conditions and having randomly distributed defects, as shown in Fig. 1. A defect is a non-active site, i.e., a site that possesses no dynamics. The density of defects in the lattice is characterized in terms of the minimum Euclidean distance d between defects. The dynamics of the diffusively coupled map system is described by

$$x_i(t+1) = (1-\epsilon) f(x_i(t)) + \frac{\epsilon}{\mathcal{N}_i} \sum_{j \in \nu_i} f(x_j(t)) \,, \tag{0.1}$$

where $x_i(t)$ is the state of an active site i ($i = 1, \ldots, N$) at time t, ν_i is the set of the nearest neighbors of site i and $\mathcal{N}_i \in \{1, 2, 3, 4\}$ is the cardinality of this set, ϵ measures the coupling

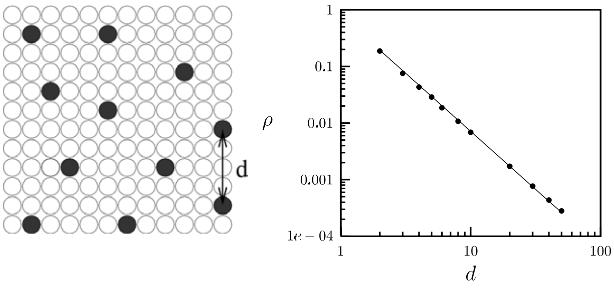

Fig. 1. (Left) spatial support of the system, showing active sites (∘) and defects (•). Defects are randomly placed in such a way that their density distribution is maximum for a given minimum distance d between defects. (Right) the defect density scales as $\rho = 0.625d^{-2}$.

strength, and $f(x(t))$ is a chaotic map that expresses the local bistable dynamics [7,8],

$$f(x) = \begin{cases} -2\mu/3 - \mu x, & \text{if } x \in [-1, -1/3] \\ \mu x, & \text{if } x \in [-1/3, 1/3] \\ 2\mu/3 - \mu x, & \text{if } x \in [1/3, 1]\ . \end{cases} \tag{0.2}$$

For $\mu \in (1, 2)$ the map has two symmetric chaotic band attractors, one with values $x_i(t) > 0$ and the other with $x_i(t) < 0$, separated by a gap about the origin. Then the local states have two well defined symmetric phases that can be characterized by spin variables defined as the sign of the state at time t, $\sigma_i(t) = \text{sign}(x_i(t))$.

We fix the local parameter at $\mu = 1.9$ and set the initial conditions as follows: one half of the active sites are randomly chosen and assigned random values uniformly distributed on the positive attractor while the other half are similarly assigned values on the negative attractor. If the number of active sites is odd, then the state of the remaining site is assigned at random on either attractor.

In regular lattices ($\rho = 0$) phase growth occurs for values $\epsilon > \epsilon_o$, where $\epsilon_o = 0.67$ [8]. In contrast, in the defective medium there exist a minimum value of ρ for which the domains formed by the two phases reach a frozen configuration for all values of the coupling ϵ. To characterize the phase ordering properties of the system Eq. (0.1) we use the normalized size of the phase domains, averaged over 50 realizations, as an order parameter given by

$$R = \lim_{t\to\infty} \frac{1}{N} \sum_{r=1}^{L/2} \sum_{i,j} \delta_{r_{ij},r} \delta_{\sigma_i(t),\sigma_j(t)}\ , \tag{0.3}$$

where r_{ij} is the Euclidean distance between nodes i and j. Figure 2 (left) shows the variation of R in the space of parameters (ϵ, d). Note that the system is heterogeneous ($R \to 0$), with coexistence of the two phases for values $\epsilon < \epsilon_0$. When $\epsilon > \epsilon_0$ two regions appear in the

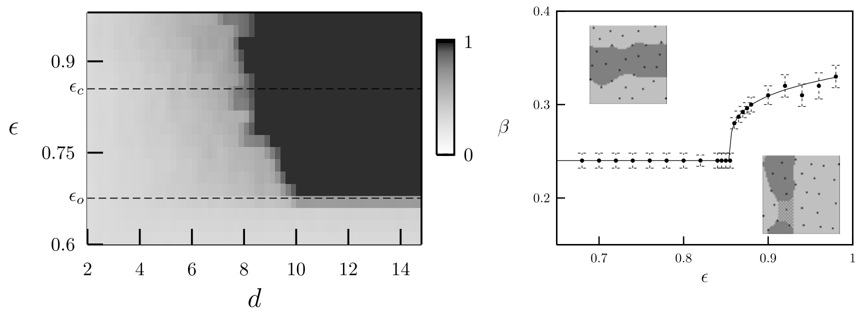

Fig. 2. (Left) normalized domain size R as a function of ϵ and d. Dark color represents homogeneous (single-phase) states and light color corresponds to heterogeneous (two-phase) states. (Right) critical exponent β vs. coupling ϵ. The two inserts show typical frozen configurations of the system for $\epsilon < \epsilon_c$ (left) and for $\epsilon > \epsilon_c$ (right). Dark and light colors represent each phase and black dots correspond to defects.

phase diagram: an heterogeneous regime for small values of d (large ρ) and an single-phase, homogeneous state ($R = 1$), for large values of d (small ρ).

The transition between these two types of behaviors induced by the presence of defects for a given value of $\epsilon > \epsilon_0$ can be described by the scaling relation $R \sim (d_c - d)^{\beta(\epsilon)}$, where $d_c(\epsilon)$ is the threshold value of the minimum distance at which the transition occurs, and β is a critical exponent.

Figure 2 (right) shows how the behavior of the critical exponent β varies with ϵ. There is a critical value of the coupling $\epsilon_c \approx 0.855$ at which β changes from being a constant value of $\beta \approx 0.24$ to follow a scaling relation $\beta \sim (\epsilon - \epsilon_c)^\gamma$, where $\gamma \approx 0.25$ is a critical exponent characterizing the transition. This transition is related to the emergence of a new type of configuration of domains induced by defects in the system, where coexistence of alternating sites of the two phases takes place forming a chessboard pattern. It should be noticed that continuous variation of the values of critical exponents may also occur in some statistical mechanics models showing phase separation dynamics [13, 14].

In summary, we have found that spatial defects can induce the formation of domains in bistable spatiotemporal systems. The minimum distance between defects acts as a parameter for the transition from a homogeneous to a heterogeneous regime. The critical exponent of this transition β exhibits a second order transition when the coupling is increased, indicating the presence of a new class of domain where both phases alternatively coexist.

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