Chaotic synchronization induced by asymmetry in coupled maps on random networks

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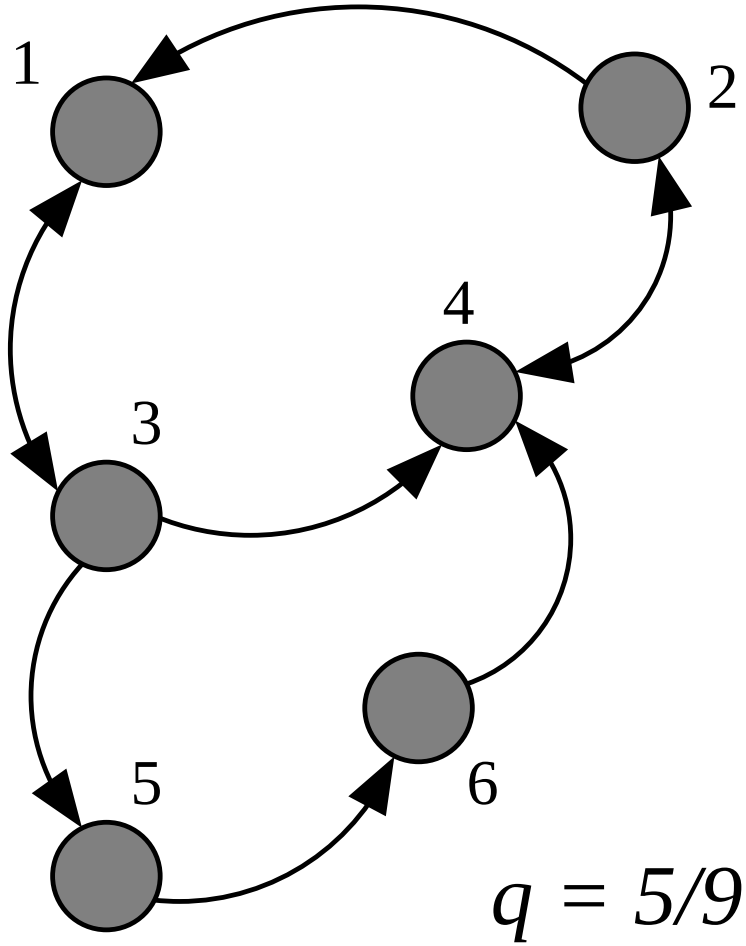
Abstract

The synchronization phenomenon occurring in asymmetrically coupled chaotic maps on a random network is studied. The asymmetry degree of network is characterized by the fraction of directed links. It is found that asymmetry induces chaotic synchronization in the system. In addition, the system undergoes a transition from an asynchronous phase to a synchronous one at some critical values of its parameters. The critical boundary separating the synchronous from the asynchronous regime is calculated on the parameter space of the system, given by the coupling strength and the asymmetry degree of the network. The phase transition between the two regimes is of second order, and the critical exponent depends of the asymmetry degree.

The Network

$$M = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

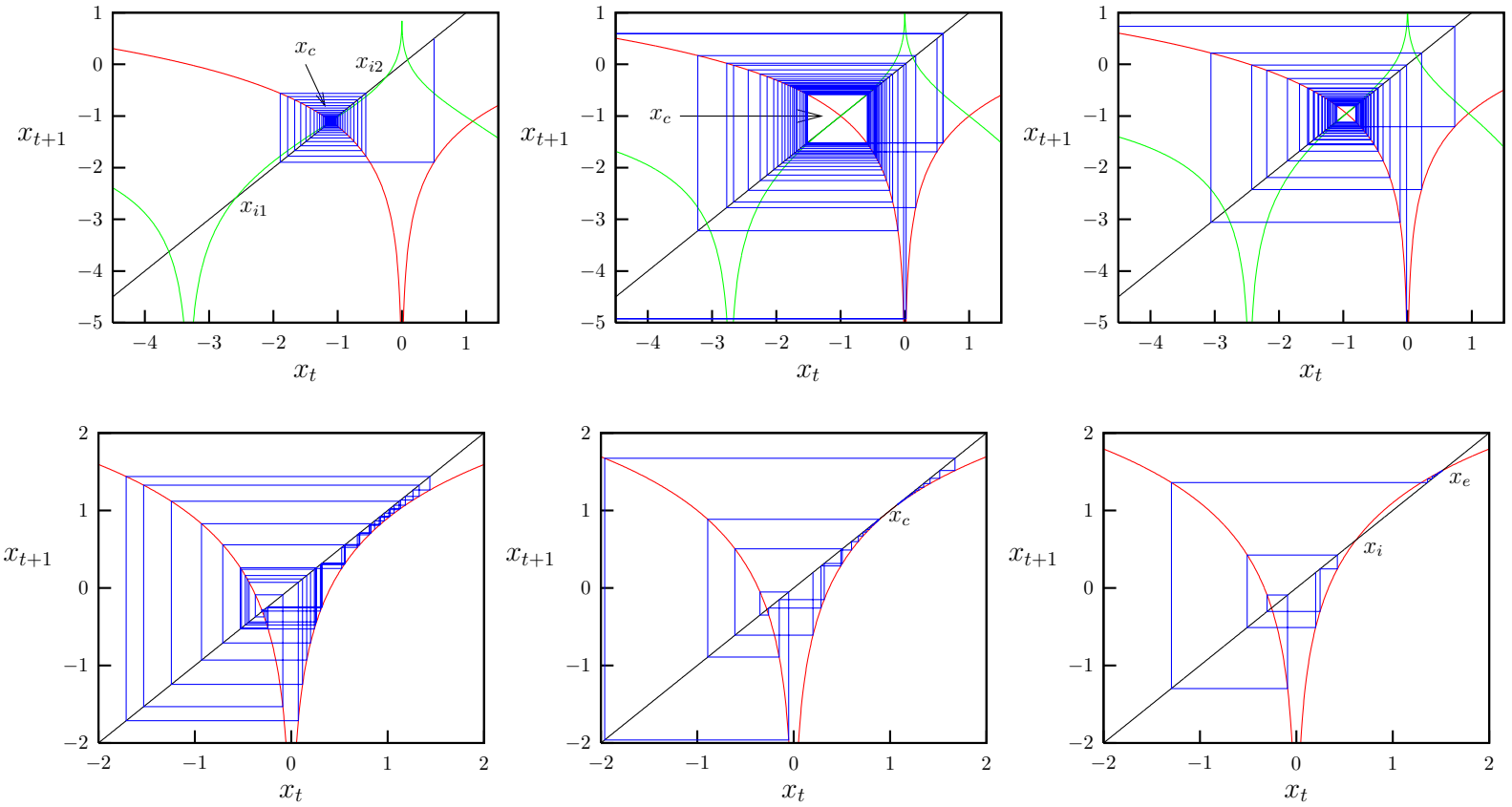


Erdõs -Rényi Random Network

$$N = 10^4$$
$$\bar{k} = 9$$

The Local Dynamic

$$x_{t+1} = b + \ln(x_t)\ , \quad b \in (-1, 1) \rightarrow \text{Robust Chaos}$$



The Model

$$x^i_{t+1} = (1-\epsilon)(b+\ln(x^i_t)) + \frac{\epsilon}{k_i}\sum_{j\in\nu_i} b+\ln(x^j_t) \quad , \quad i = 1, 2, \ldots, N$$

x^i_t : state of the ith element at discrete time t,

ϵ : coupling strength

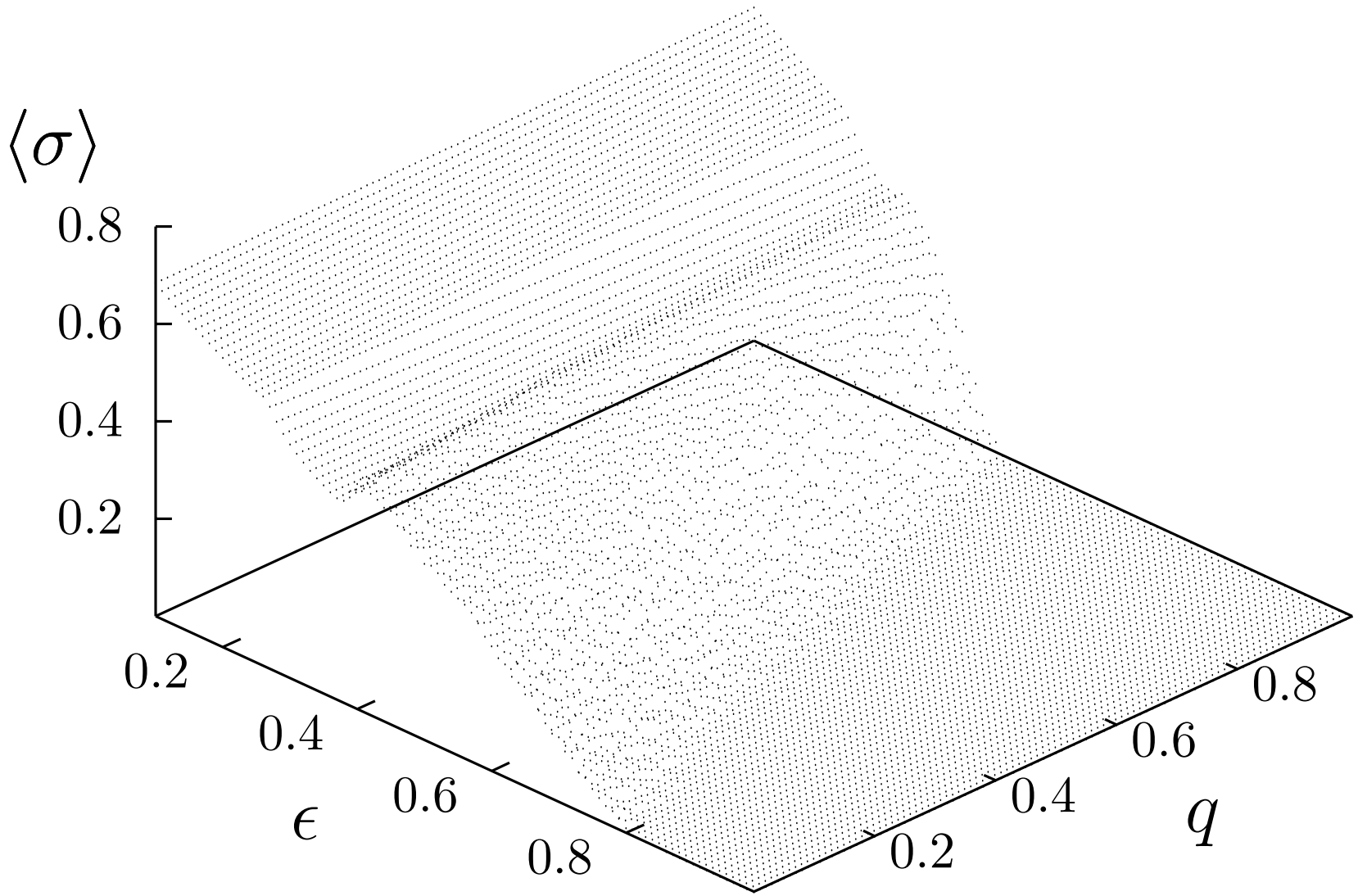
$b = -0.7$: logarithmic map parameter

ν_i : set of neighbors of the ith element

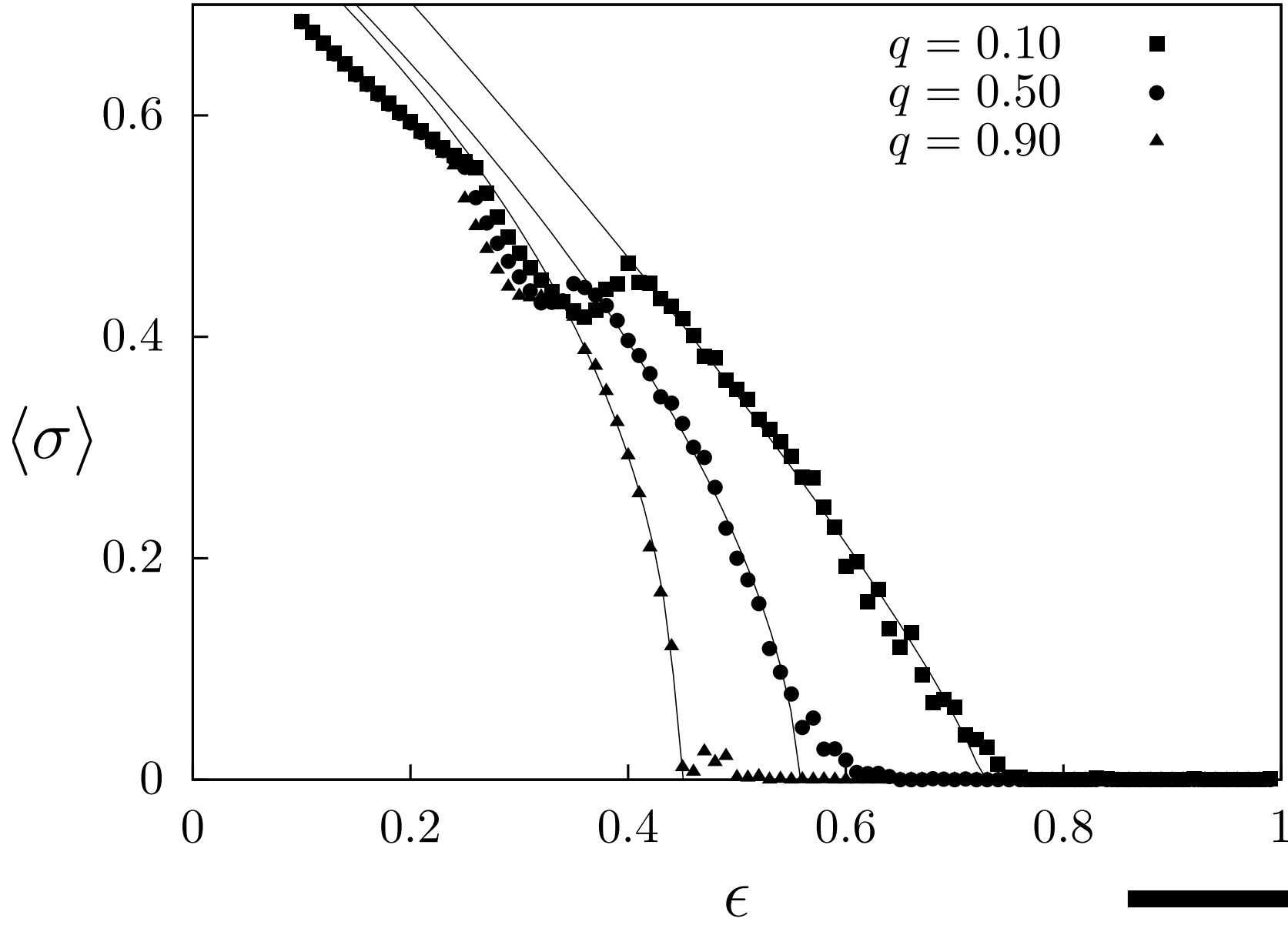
k_i : cardinality of ν_i

$$\bar{k} = \sum_{i=1}^{N} \frac{k_i}{N} \quad ; \quad \sigma_t = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x^i_t - \bar{x}_t)^2} \quad ; \quad \langle\sigma\rangle = \frac{1}{T}\sum_{t=10^4}^{10^4+T}\sigma_t$$

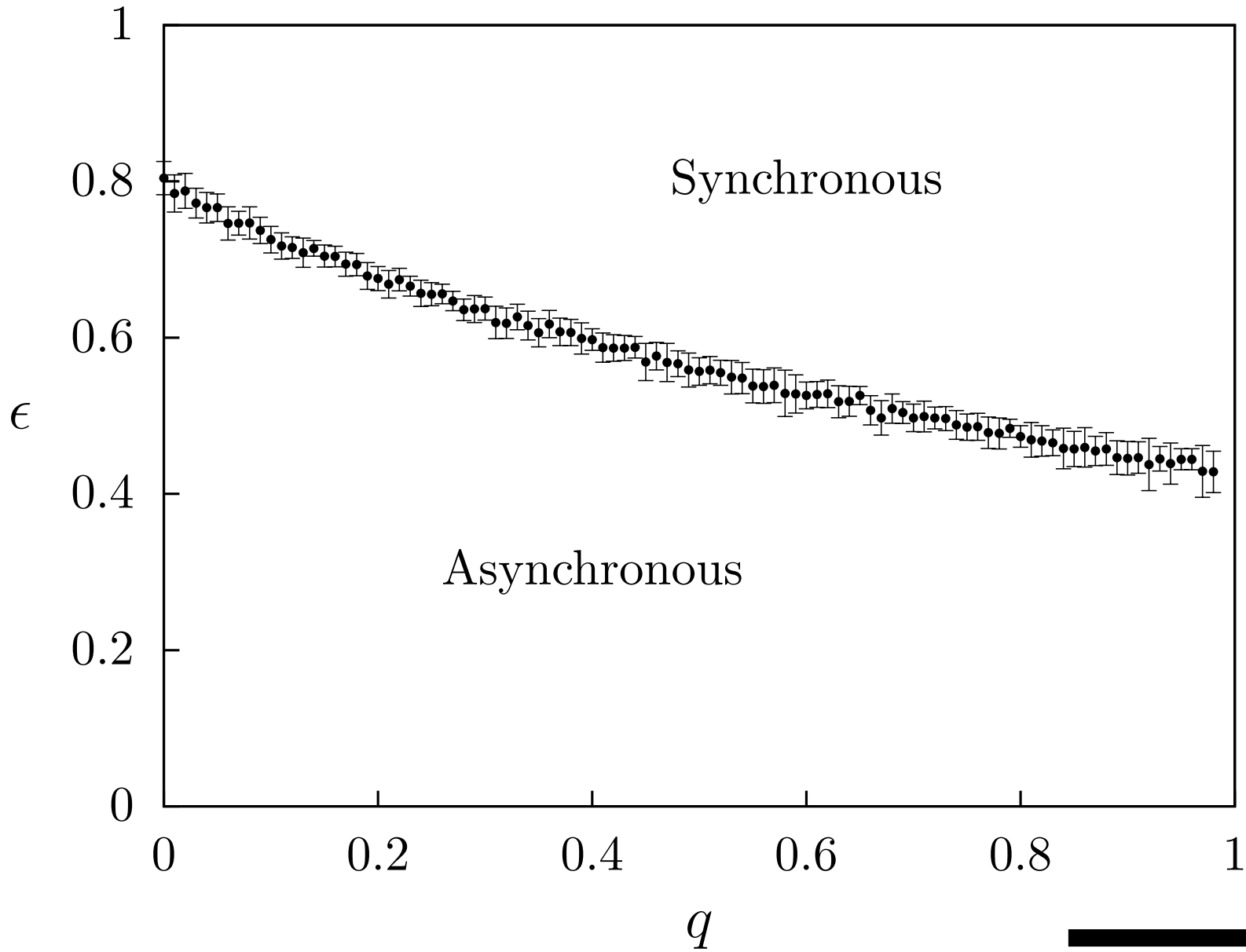
Results



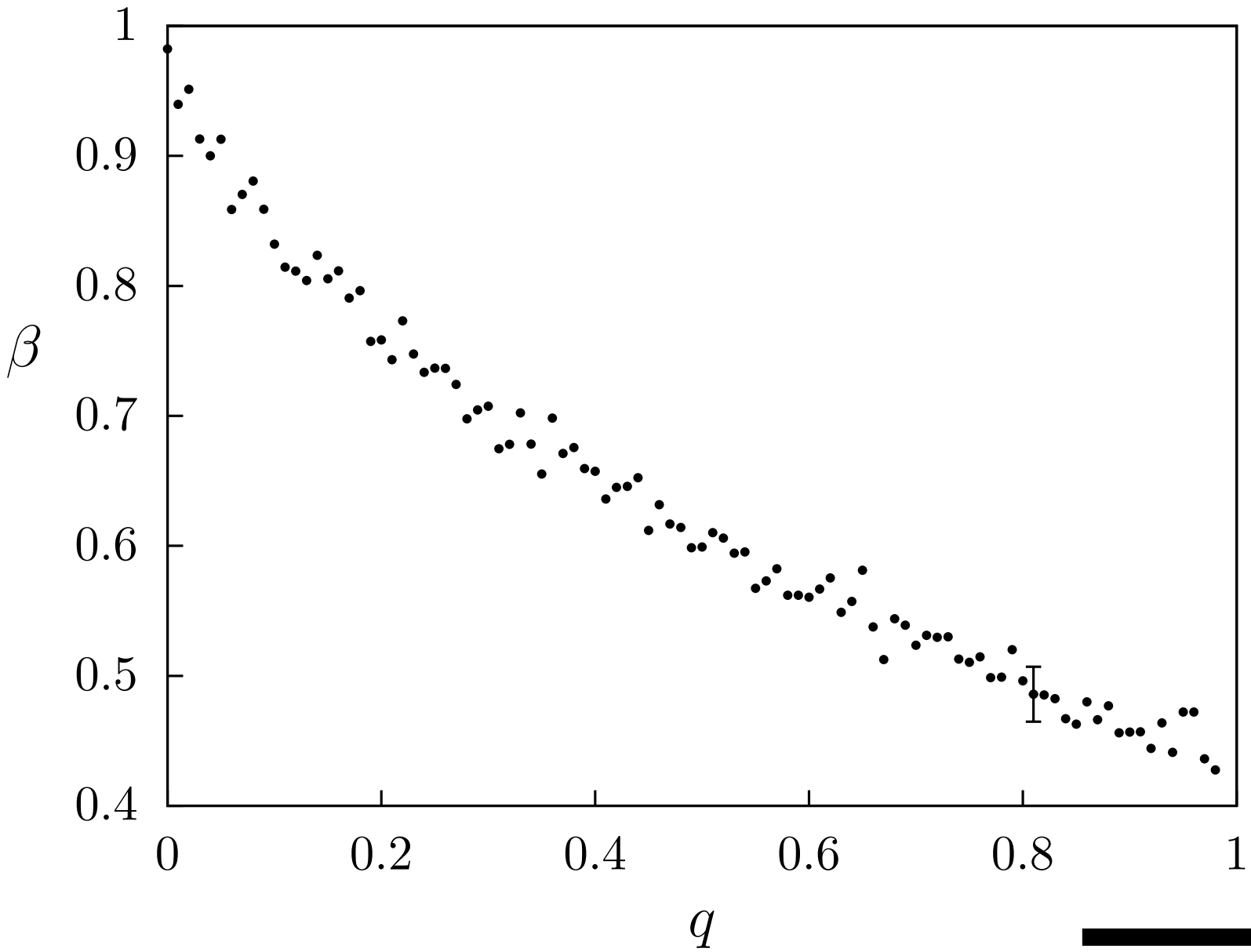
Results



Results



Results



Conclusion

- The form how a phenomena appear can depend on properties of the substrate over which the dynamic is carried out. In this case, the asynchronous-synchronous phase transition on a Coupled Chaotic Maps Lattice depends of the asymmetry degree of the network.
- In order to the synchronization phenomenon arises in the system, a smaller coupling strength, ϵ, is required when the asymmetry degree of the network, q, increases; and the phase transition becomes sharper.
- It is found a second order phase transition for all values of the asymmetry degree of the network.