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Random global coupling induces synchronization and nontrivial collective behavior in networks of chaotic maps

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Abstract. The phenomena of synchronization and nontrivial collective behavior are studied in a model of coupled chaotic maps with random global coupling. The mean field of the system is coupled to a fraction of elements randomly chosen at any given time. It is shown that the reinjection of the mean field to a fraction of randomly selected elements can induce synchronization and nontrivial collective behavior in the system. The regions where these collective states emerge on the space of parameters of the system are calculated.

There is much current interest in the investigation of collective properties of complex networks of interacting nonlinear elements [1]. As a model to study some minimal conditions for the emergence of collective behavior in a chaotic network, we consider the coupled map system

$$x^i_{t+1} = \begin{cases} (1-\epsilon)f(x^i_t) + \epsilon h_t, & \text{with probability } p, \\ f(x^i_t), & \text{with probability } 1-p, \end{cases} \tag{0.1}$$

where x^i_t ($i = 1, 2, \ldots, N$; $N =$ system size) gives the state of the ith element at discrete time t; ϵ is the coupling strength, f is a map defining the local dynamics, and

$$h_t = \frac{1}{N}\sum_{i=1}^{N} f(x^i_t) \tag{0.2}$$

is the instantaneous mean field of the system that provides a global coupling. The parameter p is the probability of connection of an element to the mean field at time t. Thus the average fraction of globally connected elements at any given time is p.

As local dynamics in Eq. (0.1) we shall consider the logarithmic map $f(x) = b + \log|x|$ [2], where b is a real parameter and $x \in (-\infty, \infty)$. This map exhibits robust chaos, with no periodic windows and no separated chaotic bands, in the parameter interval $b \in [-1, 1]$.

A synchronized state at time t is defined by the condition $x^i_t = x^j_t, \forall i, j$, in which case the dynamics is described by the single map $x_{t+1} = f(x_t)$. The synchronization of the elements in the network can be characterized by the time-average $\langle\sigma\rangle$ of the instantaneous standard deviations σ_t of the distribution of site variables x^i_t, defined as

$$\sigma_t = \left[\frac{1}{N}\sum_{i=1}^{N}\left(f(x^i_t) - h_t\right)^2\right]^{1/2}. \tag{0.3}$$

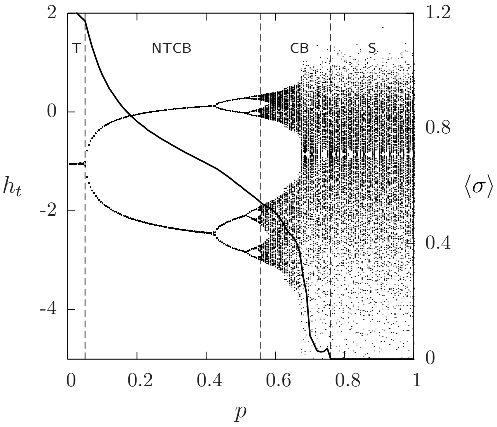

Fig. 1. Left vertical axis: bifurcation diagram of h_t as a function of p. For each value of p, the mean field was calculated at each time step during a run starting from random initial conditions on the local maps, uniformly distributed on the interval $[-8, 8]$, after discarding the transients. The regions where collective states occur are labeled T (turbulent), NTCB (nontrivial collective behavior), CB (chaotic bands), S (synchronization). Right vertical axis: $\langle\sigma\rangle$ vs p, continuous line. Fixed $b = -0.7$, $\epsilon = 0.4$; size $N = 10^5$.

Figure 1 shows the quantity $\langle\sigma\rangle$ (right vertical axis) as a function of the probability p, for fixed values of b and ϵ. There is a threshold value $p_c \simeq 0.75$ at which $\langle\sigma\rangle = 0$ (within a precision of 10^{-8} in our calculations), indicating that the elements are synchronized. The range of p where chaotic synchronization takes place is indicated by the label S. For $p = 1$, the system Eq. (0.1) becomes a globally coupled map network [3] which is known to synchronize. However, Fig. 1 reveals that the reinjection of a global coupling function to a fraction of randomly selected elements in the system is enough to achieve synchronization. The critical value p_c for the emergence of synchronization depends on the parameters of the system, as shown in Fig. 2.

The instantaneous mean field of the system h_t can characterize more complex collective behaviors arising in the system Eq. (0.1). When $b \in [-1, 1]$, the elements in the network are chaotic and desynchronized. However, for some parameter values, h_t reveals the existence of global periodic attractors. Figure 1 shows the bifurcation diagram of h_t (left vertical axis) as a function of p, for fixed b and ϵ. In this representation, collective periodic states at a given value of the parameter p appear as sets of small vertical segments which correspond to intrinsic fluctuations of the periodic orbits of the mean field.

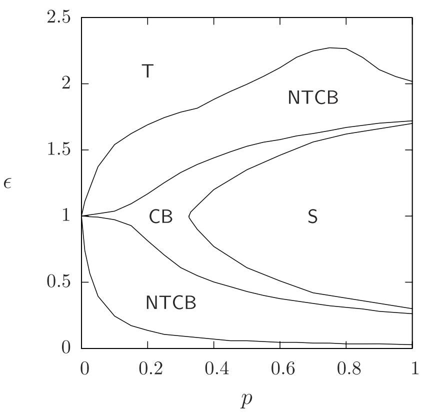

Fig. 2. Boundaries on the parameter plane (p, ϵ) separating different collective states of the system. Labels correspond to those in Fig. 1. Fixed $b = -0.7$. Size $N = 10^5$.

In the region labelled by T (turbulent) in Fig. 1, h_t follows a normal statistical behavior around a mean value (a collective fixed point) with fluctuations reflecting the averaging of N completely desynchronized chaotic elements. Increasing the probability of connection p induces a transition to periodic collective states occurring in the chaotic range of the local dynamics: a pitchfork bifurcation takes place from a collective fixed point to a collective period-two state (a state for which the time series of h_t alternatingly moves between the corresponding neighborhoods of two separate values). Collective states of higher periodicity arise by further increasing

p: global periodic attractors of period 2, 4, 8, and 16 are possible in this system. The amplitude of the collective periodic motions manifested in h_t do not decrease with an increase in the system size N. As a consequence, the variance of h_t itself does not decay as N^{-1} with increasing N but rather it saturates at some constant value related to the amplitude of the collective period. This is a phenomenon of nontrivial collective behavior, where macroscopic quantities in a spatiotemporal dynamical system exhibit ordered evolution coexisting with local chaos [4]. Note that the emergence of collective periodic behavior in this system cannot be attributed to the presence of periodic windows in the local dynamics since the logarithmic map possesses robust chaos for $b \in [-1, 1]$. Figure 1 indicates with the label NTCB the interval of p where nontrivial collective behavior arises in this system. Before crossing the boundary of the synchronization region, the collective states described by h_t take the form of chaotic bands. These states are labelled CB (collective bands) and they consist of the motion of chaotic elements that maintain some coherence.

Figure 2 shows the regions where the different collective states of the system Eq. (0.1) occur on the space of parameters (p, ϵ). These regions are separated by stability boundaries which have been numerically calculated. For $p = 1$, Fig. 2 yields the intervals of stability of the collective states S, NTCB and CB corresponding to globally coupled logarithmic maps and which have been previously found [5]. Figure 2 shows that those collective states can also emerge when a fraction of randomly selected maps are connected to the mean field and appropriate values of the coupling strength are employed.

In summary, we have shown that a reinjection of a global coupling function to a fraction of randomly selected chaotic elements in a dynamical network can induce synchronization and nontrivial collective behavior in the system. This procedure may have practical applications in the control of spatiotemporal systems. These results may be relevant in some biological and social contexts where the global information is often available only to a portion of agents in those systems.

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