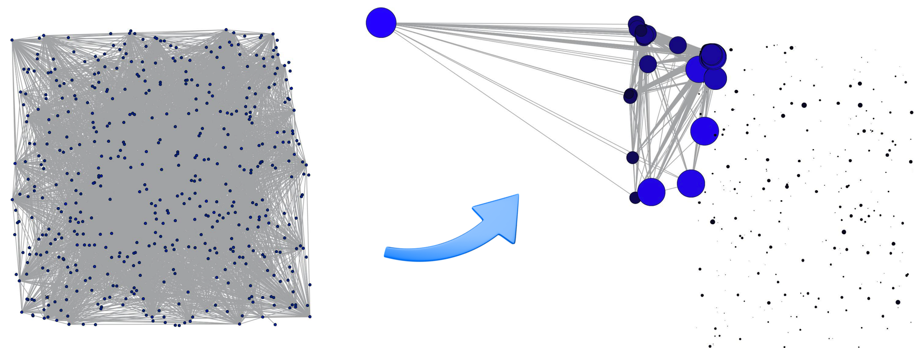

COEVOLUTIVE MODEL OF WEALTH INTERCHANGE

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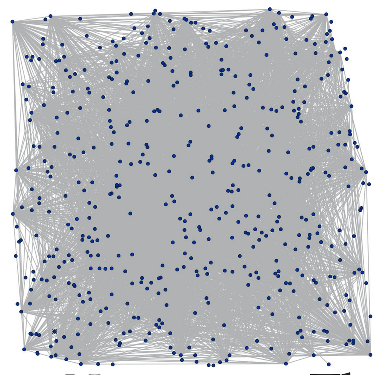
ECONOMIC MODEL

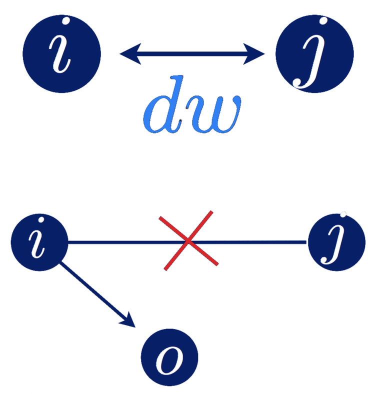
We present a model in which we investigate the structure and evolution of a random network that connects agents capable of exchanging wealth. Economic interactions between neighbors can occur only if the difference between their wealth is less than a threshold value that defines the width of the economic classes. If the interchange of wealth cannot be done, agents are reconnected with another randomly selected agent, allowing the network to evolve in time. On each interaction there is a probability of favoring the poorer agent, simulating the action of the government.

We measure the Gini index, having real world values attached to reality. Besides the network structure showed a very close connection with the economic dynamic of the system.

NETWORK DYNAMICS



- Simulations start over a random network. The system follows interaction rules of Herrera *et al* [1] model based on Laguna *et al* [2] work.
- N agents. The $i-$th agent has wealth w_i and risk aversion β_i. Each agent i is connected with a set of neighbor agents η_i.
- At each instant of time t a randomly selected agent i and an agent $j \in \eta_i$ are chosen to exchange resources.



- If $|w_i(t) - w_j(t)| < u$, wealth transfer is performed. Otherwise there is no interaction and the i-agent is disconnected from j and connected with another randomly chosen agent.
- No agent can gain more than it invests. $\rightarrow dw = \min\left[(1-\beta_i)w_i, (1-\beta_j)w_j\right]$.
- There is a probability p to favor the poorer agent that simulates the action of the government.

$$p = \frac{1}{2} + f \times \frac{|w_i(t) - w_j(t)|}{w_i(t) + w_j(t)} \geq 1/2 \qquad f \in [0, 1/2]$$

ORDER PARAMETERS

Gini index $\mathcal{G}$.

Measures the degree of inequality in the distribution of people income in a country. A Gini index of 0 represents perfect equality, while an index of 1 implies perfect inequality.

Activity $\mathcal{A}$.

Based on Herrera *et al* [1] we propose a measure of the average wealth changes on every element over time:

$$\mathcal{A} = \frac{1}{N}\sum_{t=1}^{T}\sum_{i=1}^{N}|w_t^i - w_{t-1}^i|$$

Modularity.

Measures how well a network decomposes into modular communities. A high modularity score indicates sophisticated internal structure.

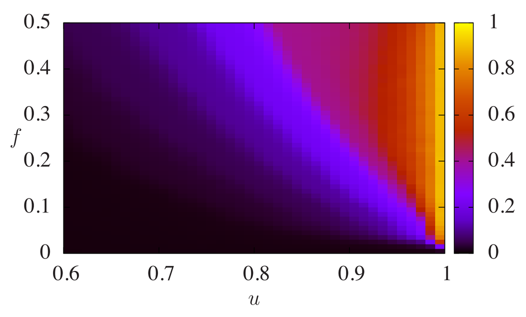
$$Q = \frac{1}{2m}\sum_{i,j}\left[a_{ij} - \frac{k_i k_j}{2m}\right]\delta(c_i, c_j)$$

where m is the total number of links. c_i is the i-node's community and δ is the Kronecker delta. $Q \in [-1, 1]$.

INITIAL CONDITIONS

All agents are over a Erdös-Rényi random network with $N = 10^4$ agents.

Each agent has $k = 16$ neighbors, an initial wealth $w_i(0) = 1\,\forall\, i$ and a risk aversion $\beta_i \in [0, 1]$ uniformly distributed over all the agents.



Activity

Simulations were made for different values of $f \in [0, 0.5]$ and $u \in [0.6, 1]$. This range of parameters obeys non nule values of the activity.

REFERENCES

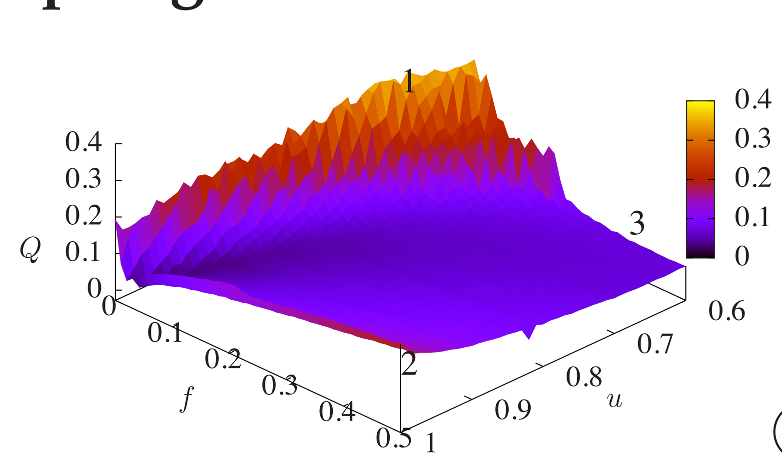
[1] Herrera J. L., Cosenza M. G., Tucci K. "A model of economic exchange in a stratified society with local interactions". Revista Científica UNET, vol. 21, p. 8. 2009.

[2] Laguna M.F., Gusman S.R., Iglesias J.R. "Economic exchanges in a stratified society: End of the middle class?" Quantitative finance papers, arXiv.org, 2005.

[3] NWB Team. "Network Workbench Tool". Indiana University, Northeastern University, and University of Michigan, http://nwb.slis.indiana.edu. 2006.

RESULTS

•Topological characterization of the economic network.



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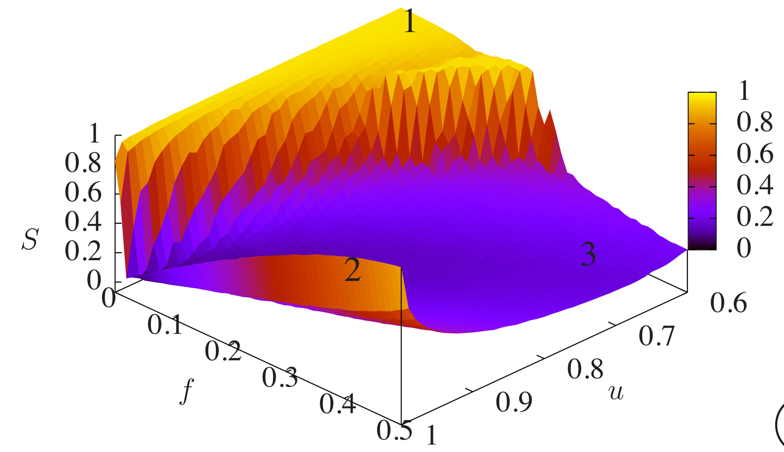

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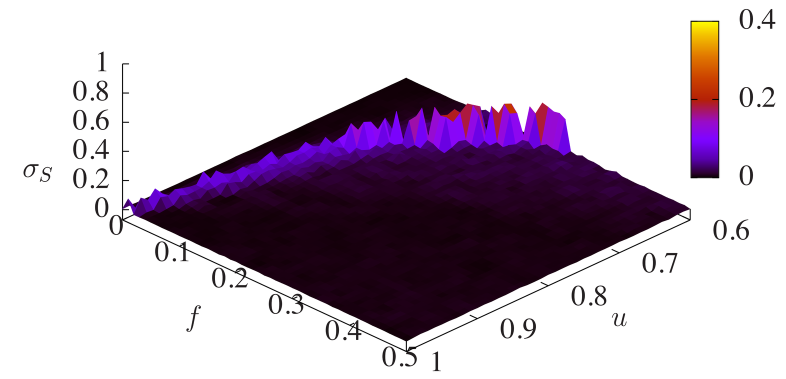
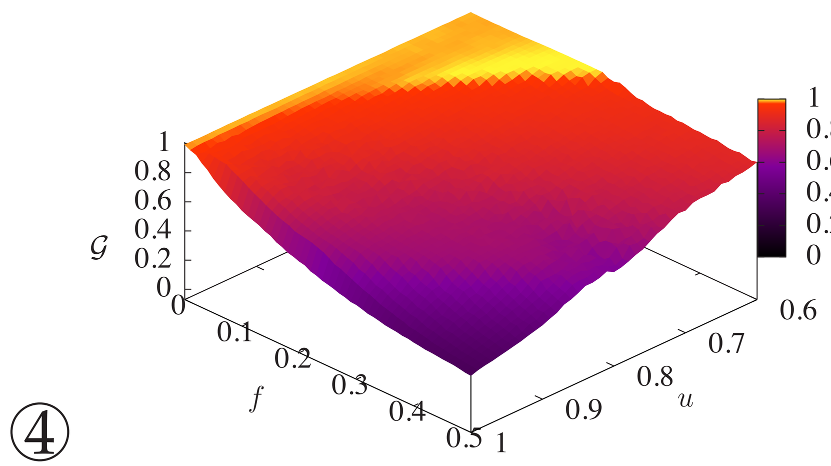


③

Figure ① unveils the existence of two regions where modularity value is significantly large. On the other side, Figure ② shows a similar behavior, two regions where the biggest subgraph is as big as the entire network ($S \approx 1$).

Standard deviation on figure ③ matchs almost perfectly with the boundary between high and low values of S. This suggests a phase transition.

•Influence of the topology on network dynamics.



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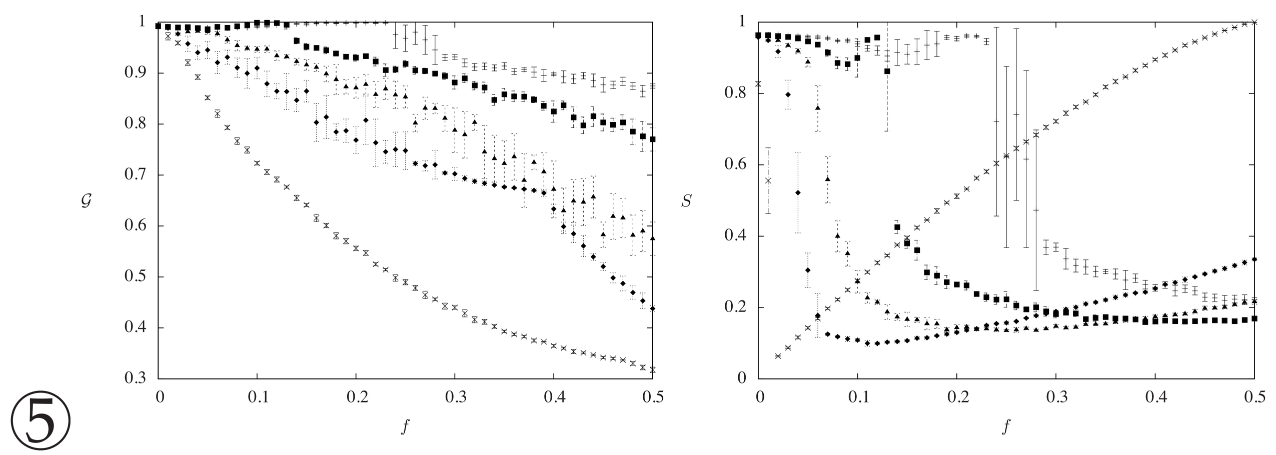
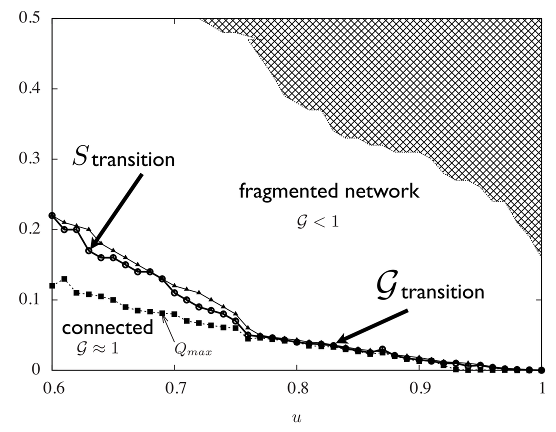


⑤

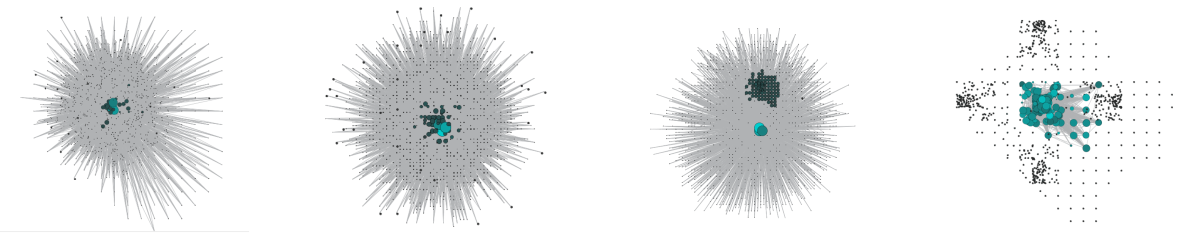
As shown in figure ④, Gini index is maximum for low f and takes real world values when $u \gtrsim 0.75$ and $f \gtrsim 0.2$. Is noticeable a decrease of $\mathcal{G}$ when u and f increase. This behavior is similar to that seen in Q and S, so we could think in a change of Gini values product of the network topology. Figure ⑤ shows $\mathcal{G}$ and S for different values of u and both match on the decay points.



$\mathcal{G}$ and S phase diagram

This phase diagram makes very clear the relation between network fragmentation and $\mathcal{G}$. The transitions match exactly. The breaking of the graph into smaller communities stimulates a Gini index reduction.

COMMUNITY STRUCTURE



Connected Transition Fragmented Real values

With the help of Network Workbench [3] we were able to graphically reproduce the network structure on each stage of the phase diagram. This is a network sample of $N = 1000$ agents. Color represents wealth and size represents connectivity.

CONCLUSIONS

The increase of f caused as expected a fairer system. We detect a critical value of f for the transition of S that varies with u. Modularity showed community structure between the transition from connected to fragmented network. And the most important result was that for all values of u, the Gini index showed an abrupt decrease directly relationed with the size of the biggest subgraph. This suggests that formation of communities breaking the network produces a fairer system.